



Figure 1

Figure 1 shows a sector  $OABCO$  of a circle centre  $O$ .

Given that

- $OA = OC = 12$  cm
- angle  $AOC = \theta$  radians
- area triangle  $OAC$  : area segment  $ABC = 3 : 1$

(a) show that

$$3\theta - 4\sin\theta = 0 \quad (2)$$

(b) Taking 1.2 as a first approximation to  $\theta$ , apply the Newton-Raphson method once to

$$f(\theta) = 3\theta - 4\sin\theta$$

to find a second approximation to  $\theta$

Give your answer to 3 decimal places.

(3)

(a) contd.  $72\sin\theta : 72\theta - 72\sin\theta = 3 : 1$

$$\Rightarrow \frac{72\sin\theta}{72\theta - 72\sin\theta} = \frac{3}{1} \Rightarrow \frac{\sin\theta}{\theta - \sin\theta} = 3 \quad (1 \text{ mark})$$

$$\Rightarrow \sin\theta = 3\theta - 3\sin\theta \Rightarrow 3\theta - 4\sin\theta = 0 \quad (1 \text{ mark})$$

(b)  $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$   $f(\theta) = 3\theta - 4\sin\theta$   
 $f'(\theta) = 3 - 4\cos\theta$  (1 mark)

$$\theta_1 = 1.2 \text{ given} \quad \theta_2 = 1.2 - \frac{f(1.2)}{f'(1.2)} = 1.2 - \frac{3(1.2) - 4\sin(1.2)}{3 - 4\cos(1.2)} \quad (1 \text{ mark})$$

$$= 1.2826\dots = 1.283 \text{ 3dp} \quad (1 \text{ mark})$$

(a) Area Triangle  $\left(\frac{1}{2}ab\sin C\right)$   
 $= \frac{1}{2}(12)(12)\sin\theta = 72\sin\theta$

Area Segment  $\left(\frac{1}{2}r^2\theta\right)$   
 $= \text{Area Sector} - \text{Area Triangle}$   
 $= \frac{1}{2}(12)^2\theta - 72\sin\theta$   
 $= 72\theta - 72\sin\theta$