7. A ball is released from rest from a height of 5 m and bounces repeatedly on horizontal ground.

After hitting the ground for the first time, the ball rises to a maximum height of 3 m.

In a model for the motion of the ball

- the maximum height after each bounce is 60% of the previous maximum height

- the motion takes place in a vertical line

(a) Using the model

    (i) show that the maximum height after the 3rd bounce is 1.08 m,

    (ii) find the total distance the ball travels from release to when the ball hits the ground for the 5th time.

(3)

According to the model, after the ball is released, there is a limit, D metres, to the total distance the ball will travel.
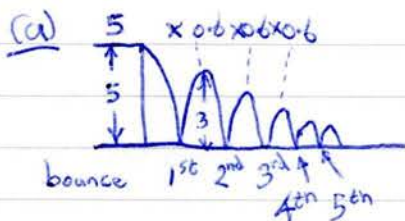
(b) Find the value of D

(2)

With reference to the model,

(c) give a reason why, in reality, the ball will not travel D metres in total.

(1)

(a)

Maximum height of each bounce is geometric sequence with $a = 3$ ← height of first bounce
$r = 0.6$

(a)(i) max. height of 3rd bounce $= ar^{n-1}$
$$= 3(0.6)^{3-1} = 3(0.6)^2 = 1.08 \text{ m} \quad (1 \text{ mark})$$

(a)(ii) motion is in a vertical line, so
total distance $= \underset{\text{initial drop}}{5} + (\underset{\text{rise \& fall for each bounce}}{2 \times S_4}) = 5 + 2\left(\dfrac{3(1-0.6^4)}{1-0.6}\right)$ (1 mark)

$$= 18.056 \text{ m} \quad (1 \text{ mark})$$

(b) total distance, $D = \underset{\text{initial drop}}{5} + \underset{\text{rise \& fall for each bounce}}{2S_\infty} = 5 + 2\left(\dfrac{3}{1-0.6}\right)$ (1 mark)

$$= 20 \text{ m} \quad (1 \text{ mark})$$

(c) • ball would not actually bounce forever, so total distance would be less than 20 m
or • ball is not a particle, but has dimensions which stop its centre from hitting ground
or • there could be horizontal motion or • air resistance, etc. (1 mark)