8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Express $3\cos x + \sin x$ in the form $R\cos(x - \alpha)$ where

- R and α are constants
- $R > 0$
- $0 < \alpha < \dfrac{\pi}{2}$

Give the exact value of R and the value of α in radians to 3 decimal places.

(3)

The temperature, θ°C, inside a rabbit hole on a particular day is modelled by the equation

$$\theta = 6.5 + 3\cos\left(\frac{\pi t}{13} - 4\right) + \sin\left(\frac{\pi t}{13} - 4\right) \qquad 0 \leqslant t < 24$$

where t is the number of hours after midnight.

Using the equation of the model and your answer to part (a)

(b) (i) deduce the minimum value of θ during this day,

(ii) find the time of day when this minimum value occurs, giving your answer to the nearest minute.

(4)

(c) Find the rate of temperature increase in the rabbit hole at midday.

(2)