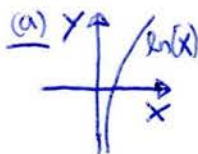


9. The function  $f$  is defined by

$$f(x) = \frac{(x+5)(x+1)}{(x+4)} - \ln(x+4)$$

$$x \in \mathbb{R} \quad x > k$$

(a) State the smallest possible value of  $k$ .



$\ln(x)$  only exists for positive  $x$ , so  
 $x > 0$   
 $x+4 > 0$   
 $x > -4$  (1 mark)

(1)

(b) Show that

$$f'(x) = \frac{ax^2 + bx + c}{(x+4)^2}$$

where  $a$ ,  $b$  and  $c$  are integers to be found.

(4)

(c) Hence show that  $f$  is an increasing function.

(2)

$$\frac{d}{dx} \left( \frac{(x+5)(x+1)}{(x+4)} \right) = \frac{d}{dx} \left( \frac{x^2 + 6x + 5}{x+4} \right)$$

Quotient Rule:  $y' = \frac{vu' - uv'}{v^2}$   
 (where  $y = \frac{u}{v}$ )

$$= \frac{(x+4)(2x+6) - (x^2 + 6x + 5)(1)}{(x+4)^2} \quad (2 \text{ marks})$$

$$= \frac{2x^2 + 14x + 24 - x^2 - 6x - 5}{(x+4)^2} = \frac{x^2 + 8x + 19}{(x+4)^2}$$

$$\frac{d}{dx} (\ln(x+4)) = \frac{1}{x+4} \times 1 = \frac{1}{x+4} \quad (1 \text{ mark})$$

$$f'(x) = \frac{d}{dx} \left( \frac{(x+5)(x+1)}{(x+4)} \right) - \frac{d}{dx} (\ln(x+4))$$

$$= \frac{x^2 + 8x + 19}{(x+4)^2} - \frac{1}{x+4} = \frac{x^2 + 8x + 19}{(x+4)^2} - \frac{x+4}{(x+4)^2}$$

$$= \frac{x^2 + 8x + 19 - x - 4}{(x+4)^2} = \frac{x^2 + 7x + 15}{(x+4)^2} \quad (1 \text{ mark})$$

(c)  $f(x)$  is an increasing function if  $f'(x)$  always  $> 0$

$$(x+4)^2 > 0 \quad \text{for } x > -4 \quad (x+4)^2 = 0 \quad \text{for } x = -4$$

$x^2 + 7x + 15$  is smiley face shape 'U' because positive coefficient of  $x^2$

Discriminant  $b^2 - 4ac = 7^2 - 4(1)(15) = 49 - 60 = -11 < 0$ , so quadratic never crosses or touches  $x$ -axis  $\uparrow \downarrow$

smiley face must be above  $x$ -axis & positive.

Numerator & Denominator of  $f'(x)$  are both positive, so  $f'(x)$  is always positive  $\Rightarrow$  increasing (2 marks)