



10.

(c) line  $l$  meets curve  $C$   
 when  $3(t^3+3t) - 5(3t^2) + 27 = 0$   
 $3t^3 + 9t - 15t^2 + 27 = 0$   
 $3t^3 - 15t^2 + 9t + 27 = 0$   
 $t^3 - 5t^2 + 3t + 9 = 0$  (1 mark)

(d) cotd  $\int_{-1}^3 9t^4 + 9t^2 dt$   
 $= \left[ \frac{9t^5}{5} + \frac{9t^3}{3} \right]_{-1}^3$   
 $= 9 \left( \frac{3^5}{5} + \frac{3^3}{3} - \frac{(-1)^5}{5} - \frac{(-1)^3}{3} \right)$   
 $= \frac{2616}{5}$  (1 mark)

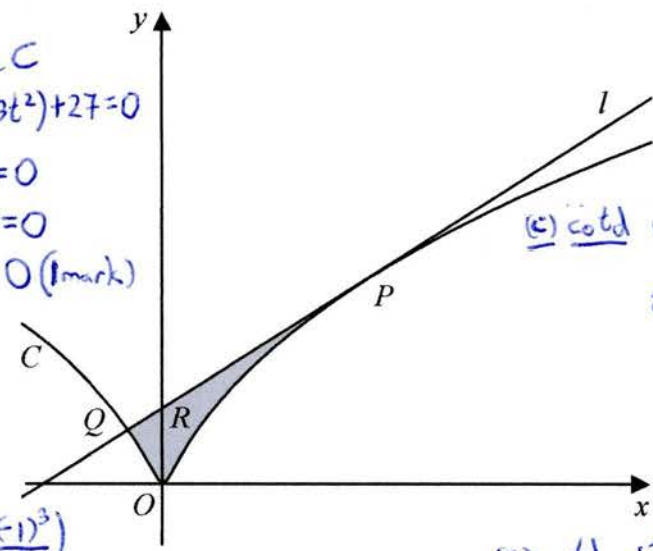


Figure 2

(c) cotd we know  $t=3$  is a solution  
 $t^2 - 2t - 3$   
 $t-3 \overline{) t^3 - 5t^2 + 3t + 9}$   
 $-(t^3 - 3t^2)$   
 $-2t^2 + 3t$   
 $-(-2t^2 + 6t)$   
 $-3t + 9$   
 $-(-3t + 9)$   
 $0$

(c) cotd  $t^2 - 2t - 3 = (t-3)(t+1)$   
 so  $t = -1$  is other  $t$  value  
 for intersection.  
 $t = -1 \Rightarrow Q$  is  $((-1)^3 + 3(-1), 3(-1)^2)$   
 $= (-4, 3)$  (2 marks)

The curve  $C$  shown in Figure 2 has parametric equations

(d) cotd Area of  $R$   
 $= 600 - \frac{2616}{5} = \frac{384}{5}$  (2 marks)  
 $x = t^3 + 3t$      $y = 3t^2$      $-2 < t < 4$

The point  $P$  lies on  $C$  where  $t = 3$  (a)  $x = (3)^3 + 3(3) = 36$

(a) Write down the coordinates of  $P$      $y = 3(3)^2 = 27$   
 $P$  is  $(36, 27)$  (1 mark)    (1)

The line  $l$  is the tangent to  $C$  at  $P$  as shown in Figure 2.

(b) Use calculus to show that an equation for  $l$  is  
 (b) cotd when  $t=3$ ,  $\frac{dy}{dx} = \frac{6(3)}{3(3)^2+3} = \frac{18}{30}$      $3x - 5y + 27 = 0$   
 $= \frac{3}{5}$  = gradient,  $m$  of line  $l$  (1 mark)    (b) cotd line  $l$  is  $\frac{y-27}{x-36} = \frac{3}{5}$     (3)

The line  $l$  meets  $C$  again at the point  $Q$      $\Rightarrow 5y - 135 = 3x - 108 \Rightarrow 3x - 5y + 27 = 0$  (2 marks)

(c) Using algebra and showing all stages of your working, find the coordinates of  $Q$     (3)

The finite region  $R$ , shown shaded in Figure 2, is bounded by the curve  $C$  and the line  $l$

(d) Using algebraic integration, find the exact area of  $R$     (5)

(d) Area  $R = \text{Area under } l - \text{Area under } C$   
 Area under  $l$   $= \frac{27+3}{2} \times 36 - (-4) = \frac{30}{2} \times 40 = 600$  (1 mark)

Area under  $C = \int_{x=-4}^{x=36} y dx = \int_{t=-1}^{t=3} y \frac{dx}{dt} dt$   
 $= \int_{t=-1}^{t=3} 3t^2(3t^2+3) dt = \int_{t=-1}^{t=3} 9t^4 + 9t^2 dt$  (1 mark)