

11. The number of bees in a colony is monitored over time.

There were 3500 bees in the colony when monitoring began.
After 1 week there were only 2000 bees in the colony.

In a simple model, the rate of decrease in the number of bees is assumed to be proportional to the square of the number of bees.

Given that there are x thousand bees in the colony t weeks after monitoring began,

(a) form and solve a differential equation to show that an equation of the model is

$$x = \frac{14}{3t + 4} \quad (6)$$

There are only 500 bees in the colony T weeks after monitoring began.

(b) Use the equation of the model to find T

(2)

(a) $\frac{dx}{dt} \propto -x^2$ $\Rightarrow \frac{dx}{dt} = -kx^2$ (1 mark)

is proportional to rate of decrease constant of proportionality

Separating the Variables, $\int \frac{1}{-kx^2} dx = \int 1 dt$ (1 mark)

$$-\frac{1}{k} \int x^{-2} dx = \int 1 dt$$

$$-\frac{1}{k} \left(\frac{x^{-1}}{-1} \right) dx = t + c \quad (\text{arbitrary constant is only needed on one side of equation})$$

$$\frac{1}{kx} = t + c \Rightarrow \frac{1}{x} = kt + kc$$

$$\frac{1}{x} = kt + c \quad (\text{arbitrary constant} \times \text{constant} = \text{arbitrary constant})$$

Given $x = 3\frac{1}{2} = \frac{7}{2}$ when $t = 0$, $\frac{1}{\frac{7}{2}} = 0 + c \Rightarrow c = \frac{2}{7}$ (1 mark)

Given $x = 2$ when $t = 1$, $\frac{1}{2} = k(1) + \frac{2}{7} \Rightarrow k = \frac{3}{14}$ (1 mark)

from above

$$\frac{1}{x} = \frac{3}{14}t + \frac{2}{7} \Rightarrow \frac{1}{x} = \frac{3t + 4}{14} \Rightarrow x = \frac{14}{3t + 4} \quad (1 \text{ mark})$$

(b) $\frac{1}{2} = \frac{14}{3T + 4} \Rightarrow 3T + 4 = 28 \Rightarrow 3T = 24$ (1 mark)

$$\Rightarrow T = 8 \text{ weeks} \quad (1 \text{ mark})$$