

12. Given that

$$y = a^x$$

where a is a positive constant

(a) prove that

$$\frac{dy}{dx} = a^x \ln a \quad (3)$$

(b) Hence show that

$$\int_1^2 4^x dx = k(\ln 2)^n$$

where k and n are integers to be found.

(3)

(a) $y = a^x$

$$\ln y = \ln(a^x)$$

$$\ln y = x \ln a$$

(1 mark)

$$\Rightarrow \ln y = (\ln a)x$$

Implicit Differentiation $\Rightarrow \frac{d(\ln y)}{dx} = \frac{d((\ln a)x)}{dx}$

Chain Rule $\rightarrow \frac{d(\ln y)}{dy} \times \frac{dy}{dx} = \ln a$ $\leftarrow \ln a$ is a constant (1 mark)

$$\frac{1}{y} \frac{dy}{dx} = \ln a \Rightarrow \frac{dy}{dx} = y \ln a = a^x \ln a$$

(1 mark)

(b) $\int_1^2 4^x dx = \frac{1}{\ln 4} \int_1^2 4^x \ln 4 dx$

$$= \frac{1}{\ln 4} [4^x]_1^2$$

(1 mark)

$$= \frac{1}{\ln 4} (4^2 - 4^1) = \frac{12}{\ln 4}$$

(1 mark)

$$= 12(\ln 4)^{-1} = 12(\ln 2^2)^{-1} = 12(2 \ln 2)^{-1}$$

$$= 12(2)^{-1} (\ln 2)^{-1} = 12\left(\frac{1}{2}\right)(\ln 2)^{-1} = 6(\ln 2)^{-1}$$

(1 mark)