

15.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

The first 3 terms of an arithmetic sequence are

$$\ln 3 \quad \ln(3^k - 1) \quad \ln(3^k + 5)$$

Find the exact value of the constant k .

(5)

Arithmetic sequence, so

common difference, $d = \ln(3^k - 1) - \ln 3 = \ln(3^k + 5) - \ln(3^k - 1)$

$$\ln\left(\frac{3^k - 1}{3}\right) = \ln\left(\frac{3^k + 5}{3^k - 1}\right)$$

$$\frac{3^k - 1}{3} = \frac{3^k + 5}{3^k - 1} \quad (1 \text{ mark})$$

$$(3^k - 1)(3^k - 1) = 3(3^k + 5)$$

$$3^{2k} - 2(3^k) + 1 = 3(3^k) + 15$$

$$3^{2k} - 5(3^k) - 14 = 0$$

$$(3^k)^2 - 5(3^k) - 14 = 0 \quad (2 \text{ marks})$$

Solving quadratic

$$(3^k + 2)(3^k - 7) = 0$$

$3^k = \cancel{-2}, 7$
↑
not possible
because
exponent > 0

so $3^k = 7 \Rightarrow k = \log_3 7$ (2 marks)