Question	Scheme	Marks	AOs
8	$y=\dfrac{(x-2)(x-4)}{4\sqrt{x}}=\dfrac{x^2-6x+8}{4\sqrt{x}}=\dfrac{1}{4}x^{\frac{3}{2}}-\dfrac{3}{2}x^{\frac{1}{2}}+2x^{-\frac{1}{2}}$	M1 A1	1.1b 1.1b
	$\displaystyle\int \frac{1}{4}x^{\frac{3}{2}}-\frac{3}{2}x^{\frac{1}{2}}+2x^{-\frac{1}{2}}\,\mathrm{d}x=\frac{1}{10}x^{\frac{5}{2}}-x^{\frac{3}{2}}+4x^{\frac{1}{2}}(+c)$	dM1 A1	3.1a 1.1b
	Deduces limits of integral are 2 and 4 and applies to their $\dfrac{1}{10}x^{\frac{5}{2}}-x^{\frac{3}{2}}+4x^{\frac{1}{2}}$	M1	2.2a
	$\left(\dfrac{32}{10}-8+8\right)-\left(\dfrac{2}{5}\sqrt{2}-2\sqrt{2}+4\sqrt{2}\right)=\dfrac{16}{5}-\dfrac{12}{5}\sqrt{2}$ Area $R=\dfrac{12}{5}\sqrt{2}-\dfrac{16}{5}\ \left(\text{or } \dfrac{16}{5}-\dfrac{12}{5}\sqrt{2}\right)$	A1	2.1
		(6)	
			(6 marks)

Notes:

M1: Correct attempt to write $\dfrac{(x-2)(x-4)}{4\sqrt{x}}$ as a sum of terms with **indices**.

Look for at least two different terms with the correct index e.g. two of $x^{\frac{3}{2}}, x^{\frac{1}{2}}, x^{-\frac{1}{2}}$ which have come from the correct places.

The correct indices may be implied later when e.g. $\sqrt{x}$ becomes $x^{\frac{1}{2}}$ or $\dfrac{1}{\sqrt{x}}$ becomes $x^{-\frac{1}{2}}$

A1: $\dfrac{1}{4}x^{\frac{3}{2}}-\dfrac{3}{2}x^{\frac{1}{2}}+2x^{-\frac{1}{2}}$ which can be left unsimplified e.g. $\dfrac{1}{4}x^{2-\frac{1}{2}}-\dfrac{1}{2}x^{\frac{1}{2}}-x^{\frac{1}{2}}+2x^{-\frac{1}{2}}$

or as e.g. $\dfrac{1}{4}\left(x^{\frac{3}{2}}-6x^{\frac{1}{2}}+8x^{-\frac{1}{2}}\right)$

The correct indices may be implied later when e.g. $\sqrt{x}$ becomes $x^{\frac{1}{2}}$ or $\dfrac{1}{\sqrt{x}}$ becomes $x^{-\frac{1}{2}}$

dM1: Integrates $x^n \to x^{n+1}$ for at least 2 correct indices

i.e. at least 2 of $x^{\frac{3}{2}}\to x^{\frac{5}{2}},\quad x^{\frac{1}{2}}\to x^{\frac{3}{2}},\quad x^{-\frac{1}{2}}\to x^{\frac{1}{2}}$

It is dependent upon the first M so at least two terms must have had a correct index.

A1: $\dfrac{1}{10}x^{\frac{5}{2}}-x^{\frac{3}{2}}+4x^{\frac{1}{2}}(+c)$. Allow unsimplified e.g. $\dfrac{1}{4}\times\dfrac{2}{5}x^{\frac{3}{2}+1}-\dfrac{1}{2}\times\dfrac{2}{3}x^{\frac{1}{2}+1}-\dfrac{2}{3}x^{\frac{1}{2}+1}+2\times 2x^{\frac{1}{2}}$

or as e.g. $\dfrac{1}{4}\left(\dfrac{2}{5}x^{\frac{5}{2}}-4x^{\frac{3}{2}}+16x^{\frac{1}{2}}\right)(+c)$.

M1: Substitutes the limits 4 and 2 to their $\dfrac{1}{10}x^{\frac{5}{2}}-x^{\frac{3}{2}}+4x^{\frac{1}{2}}$ and subtracts either way round.

There is no requirement to evaluate but 4 and 2 must be substituted either way round with evidence of subtraction, condoning omission of brackets.

E.g. condone $\dfrac{1}{10}\times 4^{\frac{5}{2}}-4^{\frac{3}{2}}+4\times 4^{\frac{1}{2}}-\dfrac{1}{10}\times 2^{\frac{5}{2}}-2^{\frac{3}{2}}+4\times 2^{\frac{1}{2}}$

This is an independent mark but the limits must be applied to an expression that is not y so they may even have differentiated.

A1: Correct working shown leading to $\frac{12}{5}\sqrt{2}-\frac{16}{5}$ but also allow $\frac{16}{5}-\frac{12}{5}\sqrt{2}$ or exact equivalents

Award this mark once one of these forms is reached and isw

See overleaf for integration by parts and integration by substitution.

Integration by parts:

$\int \frac{(x-2)(x-4)}{4\sqrt{x}}\,dx = \int \frac{1}{4}(x-2)(x-4)x^{-\frac{1}{2}}\,dx = \frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \int \frac{1}{2}(2x-6)x^{\frac{1}{2}}\,dx$	M1 A1	1.1b 1.1b
$\frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \underline{\int \frac{1}{2}(2x-6)x^{\frac{1}{2}}\,dx} = \frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \underline{\int x^{\frac{3}{2}} - 3x^{\frac{1}{2}}\,dx}$ $= \frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} \underline{- \frac{2}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}}}$ Or e.g. $= \frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \frac{1}{3}x^{\frac{3}{2}}(2x-6) + \frac{4}{15}x^{\frac{5}{2}}$	<u>dM1</u> A1	3.1a 1.1b
Deduces limits of integral are 2 and 4 and applies to their $\frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \frac{1}{3}x^{\frac{3}{2}}(2x-6) + \frac{4}{15}x^{\frac{5}{2}}$	M1	2.2a
$0 - \frac{16}{3} + \frac{128}{15} - \left(0 + \frac{4}{3}\sqrt{2} + \frac{16}{15}\sqrt{2}\right)$ Area $R = \frac{12}{5}\sqrt{2} - \frac{16}{5}$ $\left(\text{or } \frac{16}{5} - \frac{12}{5}\sqrt{2}\right)$	A1	2.1
	(6)	

Notes:

M1: Applies integration by parts and reaches the form $\alpha(x-2)(x-4)x^{\frac{1}{2}} \pm \int (px+q)x^{\frac{1}{2}}\,dx$ α, $p \neq 0$

oe e.g. $\alpha\left(x^2 - 6x + 8\right)x^{\frac{1}{2}} \pm \int (px+q)x^{\frac{1}{2}}\,dx$ α, $p \neq 0$

A1: Correct first application of parts in any form

dM1: Attempts their $\int (px+q)x^{\frac{1}{2}}\,dx$ by expanding and integrating or may attempt parts again.

E.g. $\int (2x-6)x^{\frac{1}{2}}\,dx = \int \left(2x^{\frac{3}{2}} - 6x^{\frac{1}{2}}\right)dx = \ldots$ or e.g. $\int (2x-6)x^{\frac{1}{2}}\,dx = \frac{2}{3}x^{\frac{3}{2}}(2x-6) - \frac{4}{3}\int x^{\frac{3}{2}}\,dx$

If they expand then at least one term requires $x^n \to x^{n+1}$ or if parts is attempted again, the structure must be correct.

A1: Fully correct integration in any form

M1: Substitutes the limits 4 and 2 to their $= \frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \frac{2}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}}$ and subtracts

either way round. There is no requirement to evaluate but 4 and 2 must be substituted either way round with evidence of subtraction, condoning omission of brackets.

E.g. condone $0 - \frac{16}{3} + \frac{128}{15} - 0 + \frac{4}{3}\sqrt{2} + \frac{16}{15}\sqrt{2}$

This is an independent mark but the limits must be applied to a “changed” function.

A1: Correct working shown leading to $\frac{12}{5}\sqrt{2} - \frac{16}{5}$ but also allow $\frac{16}{5} - \frac{12}{5}\sqrt{2}$ or exact equivalents

Attempts at integration by parts “the other way round” should be sent to review.

Integration by substitution example:

$u=\sqrt{x}\left(x=u^2\right)\Rightarrow\int\frac{(x-2)(x-4)}{4\sqrt{x}}\mathrm{d}x=\int\frac{\left(u^2-2\right)\left(u^2-4\right)}{4u}\frac{\mathrm{d}x}{\mathrm{d}u}\mathrm{d}u$ $=\int\frac{\left(u^2-2\right)\left(u^2-4\right)}{4u}2u\mathrm{d}u$	M1 A1	1.1b 1.1b
$=\frac{1}{2}\int\left(u^4-6u^2+8\right)\mathrm{d}u=\frac{1}{2}\left(\frac{u^5}{5}-\frac{6u^3}{3}+8u\right)(+c)$	dM1 A1	3.1a 1.1b
Deduces limits of integral are $\sqrt{2}$ and 2 and applies to their $\frac{1}{2}\left(\frac{u^5}{5}-\frac{6u^3}{3}+8u\right)$	M1	2.2a
$\frac{1}{2}\left(\frac{32}{5}-16+16-\left(\frac{4\sqrt{2}}{5}-4\sqrt{2}+8\sqrt{2}\right)\right)$ Area $R=\frac{12}{5}\sqrt{2}-\frac{16}{5}\ \left(\text{or } \frac{16}{5}-\frac{12}{5}\sqrt{2}\right)$	A1	2.1
	(6)	

Notes:

M1: Applies the substitution e.g. $u=\sqrt{x}$ and attempts $k\int\frac{\left(u^2-2\right)\left(u^2-4\right)}{u}\frac{\mathrm{d}x}{\mathrm{d}u}\mathrm{d}u$

A1: Fully correct integral in terms of u in any form e.g. $\frac{1}{2}\int\left(u^2-2\right)\left(u^2-4\right)\mathrm{d}u$

dM1: Expands the bracket and integrates $u^n\to u^{n+1}$ for at least 2 correct indices

i.e. at least 2 of $u^4\to u^5,\ u^2\to u^3,\ k\to ku$

A1: $\frac{1}{2}\left(\frac{u^5}{5}-\frac{6u^3}{3}+8u\right)(+c)$. Allow unsimplified.

M1: Substitutes the limits 2 and $\sqrt{2}$ to their $\frac{1}{2}\left(\frac{u^5}{5}-\frac{6u^3}{3}+8u\right)$ and subtracts either way round.

There is no requirement to evaluate but 2 and $\sqrt{2}$ must be substituted either way round with evidence of subtraction, condoning omission of brackets.

E.g. condone $\frac{1}{2}\left(\frac{32}{5}-16+16-\frac{4\sqrt{2}}{5}-4\sqrt{2}+8\sqrt{2}\right)$

Alternatively reverses the substitution and applies the limits 4 and 2 with the same conditions.

A1: Correct working shown leading to $\frac{12}{5}\sqrt{2}-\frac{16}{5}$ but also allow $\frac{16}{5}-\frac{12}{5}\sqrt{2}$ or exact equivalents

Award this mark once one of these forms is reached and isw.

There may be other substitutions seen and the same marking principles apply.