

Question	Scheme	Marks	AOs
11	$n(n^2 + 5)$		
	Attempts even or odd numbers Sets $n = 2k$ or $n = 2k \pm 1$ oe and attempts $n(n^2 + 5)$	M1	3.1a
	Achieves $2k(4k^2 + 5)$ (for $n = 2k$) and states “even” Or achieves $(2k + 1)(4k^2 + 4k + 6) = 2(2k + 1)(2k^2 + 2k + 3)$ (for $n = 2k + 1$) and states “even” Or e.g. achieves $(2k - 1)(4k^2 - 4k + 6) = 2(2k - 1)(2k^2 - 2k + 3)$ (for $n = 2k - 1$) and states “even”	A1	2.2a
	Attempts even and odd numbers Sets $n = 2k$ and $n = 2k \pm 1$ oe and attempts $n(n^2 + 5)$	dM1	2.1
	Achieves $2k(4k^2 + 5)$ (for $n = 2k$) and states “even” and achieves $(2k \pm 1)(4k^2 \pm 4k + 6) = 2(2k \pm 1)(2k^2 \pm 2k + 3)$ (for $n = 2k \pm 1$) and states “even” Correct work and states even for both WITH a final conclusion showing that true for all $n (\in \mathbb{N})$ or e.g. true for all even and odd numbers.	A1	2.4
		(4)	
(4 marks)			

Notes:

M1: For the key step attempting to find $n(n^2 + 5)$ when $n = 2k$ **or** $n = 2k \pm 1$ or equivalent representation of odd or even e.g. $n = 2k + 2$ **or** $n = 2k \pm 7$
Condone the use of e.g. $n = 2n$ and $n = 2n \pm 1$

A1: Achieves $2k(4k^2 + 5)$ or e.g. $2(4k^3 + 5k)$ and deduces that this is even at the appropriate time.
Or achieves $(2k \pm 1)(4k^2 \pm 4k + 6) = 2(2k \pm 1)(2k^2 \pm 2k + 3)$ oe e.g. $2(4k^3 + 6k^2 + 8k + 3)$ and deduces that this is even.

Note that if the bracket is expanded to e.g. $8k^3 + 12k^2 + 16k + 6$ then stating “even” is insufficient – they would need to say e.g. even + even + even + even = even or equivalent

Note it is also acceptable to use a divisibility argument e.g. $\frac{8k^3 + 10k}{2} = 4k^3 + 5k$ so $8k^3 + 10k$ must be even.

There should be no errors in the algebra but allow e.g. invisible brackets if they are “recovered”.

dM1: Attempts $n(n^2 + 5)$ when $n = 2k$ **and** $n = 2k \pm 1$ or equivalent representation of odd or even
e.g. $n = 2k + 2$ **and** $n = 2k \pm 7$

A1: Correct work and states even for both **WITH** a final conclusion e.g. so true for all $n (\in \mathbb{N})$.

There should be no errors in the algebra but allow e.g. invisible brackets if they are “recovered”.

A “solution” via just logic.

E.g.

If n is odd, then $n(n^2 + 5)$ is odd \times (odd + odd) = odd \times even = even

If n is even, then $n(n^2 + 5)$ is even \times (even + odd) = even \times odd = even

Both cases must be considered to score any marks and scores SC 1010 if fully correct

OR

E.g. $n(n^2 + 5) = n^3 + 5n$

If n is odd, then n^3 is odd and $5n$ is odd, so $n^3 + 5n$ is odd + odd = even

If n is even, then n^3 is even and $5n$ is even, so $n^3 + 5n$ is even + even = even

Both cases must be considered to score any marks and scores SC 1010 if fully correct

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A solution via contradiction.

M1 A1: There exists a number n such that $n(n^2 + 5)$ is odd, and so deduces that both n and $n^2 + 5$ are odd. Note that M1A0 is not possible via this method.

dM1: Sets $n^2 + 5 = 2k + 1$ (for some integer k) $\Rightarrow n^2 = 2k - 4 = 2(k - 2)$ which is even
Must use algebra here for this approach and not a “logic” argument.

A1: States that "this is a contradiction as if n^2 is even, then n is even" and then concludes so " $n(n^2 + 5)$ is even for all n ."

Attempts at proof by induction should be sent to review