Question	Scheme	Marks	AOs
12(a)	$\mathrm{f}(x)=\dfrac{\mathrm{e}^{3x}}{4x^2+k} \Rightarrow \mathrm{f}'(x)=\dfrac{\left(4x^2+k\right)3\mathrm{e}^{3x}-8x\mathrm{e}^{3x}}{\left(4x^2+k\right)^2}$ or $\mathrm{f}(x)=\mathrm{e}^{3x}\left(4x^2+k\right)^{-1} \Rightarrow \mathrm{f}'(x)=3\mathrm{e}^{3x}\left(4x^2+k\right)^{-1}-8x\mathrm{e}^{3x}\left(4x^2+k\right)^{-2}$	M1 A1	1.1b 1.1b
	$\mathrm{f}'(x)=\dfrac{\left(12x^2-8x+3k\right)\mathrm{e}^{3x}}{\left(4x^2+k\right)^2}$	A1	2.1
		(3)	
(b)	If $y=\mathrm{f}(x)$ has at least one stationary point then $12x^2-8x+3k=0$ has at least one root	B1	2.2a
	Applies $b^2-4ac\ (\geqslant)\ 0$ with $a=12, b=-8, c=3k$	M1	2.1
	$0<k\leqslant\dfrac{4}{9}$	A1	1.1b
		(3)	
			(6 marks)

Notes:

(a)

M1: Attempts the quotient rule to obtain an expression of the form $\dfrac{\alpha\left(4x^2+k\right)\mathrm{e}^{3x}-\beta x\mathrm{e}^{3x}}{\left(4x^2+k\right)^2}$, $\alpha,\beta>0$

condoning bracketing errors/omissions as long as the intention is clear.
If the quotient rule formula is quoted it must be correct.

Condone e.g. $\mathrm{f}'(x)=\dfrac{\left(4x^2+k\right)3\mathrm{e}^{3x}-8x\mathrm{e}^{3x}}{\left(4x^2+k\right)}$ provided an incorrect formula is not quoted.

May also see product rule applied to $\mathrm{e}^{3x}\left(4x^2+k\right)^{-1}$ to obtain an expression of the form

$\alpha\mathrm{e}^{3x}\left(4x^2+k\right)^{-1}+\beta x\mathrm{e}^{3x}\left(4x^2+k\right)^{-2}$ $\alpha,\beta 0<0$ condoning bracketing errors/omissions as long as the intention is clear. If the product rule formula is quoted it must be correct.

A1: Correct differentiation in any form with correct bracketing which may be implied by subsequent work.

A1: Obtains $\mathrm{f}'(x)=\left(12x^2-8x+3k\right)\mathrm{g}(x)$ where $\mathrm{g}(x)=\dfrac{\mathrm{e}^{3x}}{\left(4x^2+k\right)^2}$ **or equivalent**

e.g. $\mathrm{g}(x)=\mathrm{e}^{3x}\left(4x^2+k\right)^{-2}$

Allow recovery from "invisible" brackets earlier and apply isw here once a correct answer is seen.
Note that the complete form of the answer is not given so allow candidates to go from e.g.
$\dfrac{\left(4x^2+k\right)3\mathrm{e}^{3x}-8x\mathrm{e}^{3x}}{\left(4x^2+k\right)^2}$ or $3\mathrm{e}^{3x}\left(4x^2+k\right)^{-1}-8x\mathrm{e}^{3x}\left(4x^2+k\right)^{-2}$ to $\dfrac{\left(12x^2-8x+3k\right)\mathrm{e}^{3x}}{\left(4x^2+k\right)^2}$ for the final mark.

The "$f'(x)=$" must appear at some point but allow e.g. "$\frac{dy}{dx}=$"

(b) Note that B0M1A1 is not possible in (b)

B1: Deduces that if $y=f(x)$ has at least one stationary point then $12x^2-8x+3k=0$ has at least one root. There is no requirement to formally state $\frac{e^{3x}}{\left(4x^2+k\right)^2}>0$

This may be implied by an attempt at $b^2-4ac\geqslant 0$ or $b^2-4ac>0$ condoning slips.

M1: Attempts $b^2-4ac...0$ with $a=12,\ b=-8,\ c=3k$ where … is e.g. "=", <, >, etc.

Alternatively attempts to complete the square and sets rhs ...0

E.g. $12x^2-8x+3k=0\Rightarrow x^2-\frac{2}{3}x+\frac{1}{4}k=0\Rightarrow\left(x-\frac{1}{3}\right)^2=\frac{1}{9}-\frac{1}{4}k$ leading to $\frac{1}{9}-\frac{1}{4}k\geqslant 0$

A1: $0<k\leqslant\frac{4}{9}$ but condone $k\leqslant\frac{4}{9}$ and condone $0\leqslant k\leqslant\frac{4}{9}$

Must be in terms of k not x so do not allow e.g. $0<x\leqslant\frac{4}{9}$ but condone $\left(0,\ \frac{4}{9}\right]$ or $\left[0,\ \frac{4}{9}\right]$