Question	Scheme	Marks	AOs
15(a)	Uses the common ratio $\dfrac{5+2\sin\theta}{12\cos\theta}=\dfrac{6\tan\theta}{5+2\sin\theta}$ o.e.	M1	3.1a
	Cross multiplies and uses $\tan\theta\times\cos\theta=\sin\theta$ $(5+2\sin\theta)^2=6\times12\sin\theta$	dM1	1.1b
	Proceeds to given answer $25+20\sin\theta+4\sin^2\theta=72\sin\theta$ $\Rightarrow 4\sin^2\theta-52\sin\theta+25=0$ *	A1*	2.1
		(3)	
(a) Alt	**(a) Alternative example:**		
	Uses the common ratio $12r\cos\theta=5+2\sin\theta,\ 12r^2\cos\theta=6\tan\theta$ $\Rightarrow 12\cos\theta\left(\dfrac{5+2\sin\theta}{12\cos\theta}\right)^2=6\tan\theta$	M1	3.1a
	Multiplies up and uses $\tan\theta\times\cos\theta=\sin\theta$ $(5+2\sin\theta)^2=6\tan\theta\times12\cos\theta=72\sin\theta$	dM1	1.1b
	Proceeds to given answer $25+20\sin\theta+4\sin^2\theta=72\sin\theta$ $\Rightarrow 4\sin^2\theta-52\sin\theta+25=0$ *	A1*	2.1
		(3)	
(b)	$4\sin^2\theta-52\sin\theta+25=0\Rightarrow\sin\theta=\dfrac{1}{2}\left(,\dfrac{25}{2}\right)$	M1	1.1b
	$\theta=\dfrac{5\pi}{6}$	A1	1.2
		(2)	
(c)	Attempts a value for either a or r e.g. $a=12\cos\theta=\ 12\times-\dfrac{\sqrt{3}}{2}$ **or** $r=\dfrac{5+2\sin\theta}{12\cos\theta}=\dfrac{5+2\times\frac{1}{2}}{12\times-\frac{\sqrt{3}}{2}}$	M1	3.1a
	$"a"=-6\sqrt{3}$ **and** $"r"=-\dfrac{1}{\sqrt{3}}$ o.e.	A1	1.1b
	Uses $S_\infty=\dfrac{a}{1-r}=\dfrac{-6\sqrt{3}}{1+\frac{1}{\sqrt{3}}}$	dM1	2.1
	Rationalises denominator $S_\infty=\dfrac{-6\sqrt{3}}{1+\frac{1}{\sqrt{3}}}=\dfrac{-18}{\sqrt{3}+1}\times\dfrac{\sqrt{3}-1}{\sqrt{3}-1}$	ddM1	1.1b
	$(S_\infty=)9(1-\sqrt{3})$	A1	2.1
		(5)	
			(10 marks)

Notes:

(a)

M1: For the key step in using the ratio of $\dfrac{a_2}{a_1}=\dfrac{a_3}{a_2}$

dM1: Cross multiplies and uses $\tan\theta\times\cos\theta=\sin\theta$

A1*: Proceeds to the given answer including the " = 0" with no errors and sufficient working shown.

Alternative:

M1: Expresses the 2nd and 3rd terms in terms of the first term and the common ratio and eliminates "r"

dM1: Multiplies up and uses $\tan\theta \times \cos\theta = \sin\theta$

A1*: Proceeds to the given answer including the " = 0" with no errors and sufficient working shown.

Other approaches may be seen in (a) and can be marked in a similar way e.g. M1 for correctly obtaining an equation in θ using the GP, M1 for applying $\tan\theta \times \cos\theta = \sin\theta$ or equivalent and eliminating fractions, A1 as above

Example: $u_2 = \dfrac{u_1 \times u_3}{u_2} \Rightarrow 5 + 2\sin\theta = \dfrac{12\cos\theta \times 6\tan\theta}{5 + 2\sin\theta}$ **M1**

$\Rightarrow (5 + 2\sin\theta)^2 = 72\sin\theta$ **dM1**

$25 + 20\sin\theta + 4\sin^2\theta = 72\sin\theta$

$\Rightarrow 4\sin^2\theta - 52\sin\theta + 25 = 0$ * **A1**

(b)

M1: Attempts to solve $4\sin^2\theta - 52\sin\theta + 25 = 0$. Must be clear they have found $\sin\theta$ and not e.g. just x from $4x^2 - 52x + 25 = 0$. Working does not need to be seen but see general guidance for solving a 3TQ if necessary. Note that the $\dfrac{25}{2}$ does not need to be seen.

A1: $\theta = \dfrac{5\pi}{6}$ and no other values unless they are rejected or the $\dfrac{5\pi}{6}$ clearly selected here and not in (c)

A minimum requirement in (b) is e.g. $\sin\theta = \dfrac{1}{2}, \quad \theta = \dfrac{5\pi}{6}$

Do **not** allow 150° for $\dfrac{5\pi}{6}$

PTO for the notes to part (c)

(c) Allow full marks in (c) if e.g. $\theta=\frac{\pi}{6}$ is their answer to (b) but $\theta=\frac{5\pi}{6}$ is used here.

or if e.g. $\theta=\frac{5\pi}{6}$ is their answer to (b) but $\theta=\frac{\pi}{6}$ is used here allow the M marks only.

M1: For attempting a value (exact or decimal) for either a or r using **their** θ

E.g. $a=12\cos\theta=\left(12\times-\frac{\sqrt{3}}{2}\right)$ or $r=\frac{5+2\sin\theta}{12\cos\theta}=\left(\frac{5+2\times\frac{1}{2}}{12\times-\frac{\sqrt{3}}{2}}\right)$ oe e.g. $r=\frac{6\tan\theta}{5+2\sin\theta}=\left(\frac{6\times-\frac{1}{\sqrt{3}}}{5+2\times\frac{1}{2}}\right)$

A1: Finds both $a=-6\sqrt{3}$ and $r=-\frac{1}{\sqrt{3}}$ which can be left unsimplified but $\sin\theta=\frac{1}{2},\cos\theta=-\frac{\sqrt{3}}{2}$

and $\tan\theta=-\frac{\sqrt{3}}{3}$ (if required) must have been used.

dM1: Uses both **values** of "a" and "r" with the equation $S_\infty=\frac{a}{1-r}=\frac{-6\sqrt{3}}{1+\frac{1}{\sqrt{3}}}$ to create an expression

involving surds where a and r have come from appropriate work and $|r|<1$

Depends on the first method mark.

ddM1: Rationalises denominator. The denominator must be of the form $p\pm q\sqrt{3}$ oe e.g. $p+\frac{q}{\sqrt{3}}$

Depends on both previous method marks.

Note that stating e.g. $\frac{k}{p+q\sqrt{3}}\times\frac{p-q\sqrt{3}}{p-q\sqrt{3}}$ or $\frac{k}{p+\frac{q}{\sqrt{3}}}\times\frac{p-\frac{q}{\sqrt{3}}}{p-\frac{q}{\sqrt{3}}}$ is sufficient.

A1: Obtains $(S_\infty=)9(1-\sqrt{3})$

Note that full marks are available in (c) for the use of $\theta=150°$

Note also that marks may be implied in (c) by e.g.

$$S_\infty=\frac{a}{1-r}=\frac{12\cos\theta}{1-\frac{5+2\sin\theta}{12\cos\theta}}=\frac{144\cos^2\theta}{12\cos\theta-5-2\sin\theta}=\frac{144\cos^2\frac{5\pi}{6}}{12\cos\frac{5\pi}{6}-5-2\sin\frac{5\pi}{6}}$$

$$=\frac{108}{-6-6\sqrt{3}}=\frac{108}{-6-6\sqrt{3}}\times\frac{-6+6\sqrt{3}}{-6+6\sqrt{3}}=\frac{-648+648\sqrt{3}}{-72}=9(1-\sqrt{3})$$

Scores M1A1 implied dM1 ddM1 A1

See next page for some other cases in (c) and how to mark them:

$$S_\infty = \frac{a}{1-r} = \frac{12\cos\frac{5\pi}{6}}{1-\dfrac{5+2\sin\frac{5\pi}{6}}{12\cos\frac{5\pi}{6}}} \quad \text{or e.g.} \quad S_\infty = \frac{a}{1-r} = \frac{12\cos\frac{\pi}{6}}{1-\dfrac{5+2\sin\frac{\pi}{6}}{12\cos\frac{\pi}{6}}}$$

And nothing else

scores M1A0dM1ddM0A0

$$S_\infty = \frac{a}{1-r} = \frac{12\cos\frac{5\pi}{6}}{1-\dfrac{5+2\sin\frac{5\pi}{6}}{12\cos\frac{5\pi}{6}}} = 9\left(1-\sqrt{3}\right)$$

Scores M1A1dM1ddM0A0

$$S_\infty = \frac{a}{1-r} = \frac{12\cos\frac{\pi}{6}}{1-\dfrac{5+2\sin\frac{\pi}{6}}{12\cos\frac{\pi}{6}}} = 9\left(1+\sqrt{3}\right)$$

Scores M1A0dM1ddM0A0

$S_\infty = 9\left(1-\sqrt{3}\right)$ with no working scores no marks