Question	Scheme	Marks	AOs
16(a)	Attempts $\dfrac{dy}{dx}=\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}=\dfrac{4\sec^2 t\tan t}{2\sec^2 t}(=2\tan t)$	M1 A1	1.1b 1.1b
	At $t=\dfrac{\pi}{4}$, $\dfrac{dy}{dx}=2, x=3, y=7$	M1	2.1
	Attempts equation of normal $y-7=-\dfrac{1}{2}(x-3)$	M1	1.1b
	$y=-\dfrac{1}{2}x+\dfrac{17}{2}$ *	A1*	2.1
		(5)	
(b)	Attempts to use $\sec^2 t=1+\tan^2 t\Rightarrow\dfrac{y-3}{2}=1+\left(\dfrac{x-1}{2}\right)^2$	M1	3.1a
	$\Rightarrow y-3=2+\dfrac{(x-1)^2}{2}\Rightarrow y=\dfrac{1}{2}(x-1)^2+5$ *	A1*	2.1
		(2)	
	(b) Alternative 1:		
	$y=\dfrac{1}{2}(x-1)^2+5=\dfrac{1}{2}(2\tan t+1-1)^2+5$ $=\dfrac{1}{2}4\tan^2 t+5=2\left(\sec^2 t-1\right)+5$	M1	3.1a
	$=2\sec^2 t+3=y$*	A1	2.1
	(b) Alternative 2:		
	$x=2\tan t+1\Rightarrow t=\tan^{-1}\left(\dfrac{x-1}{2}\right)\Rightarrow y=2\sec^2\left(\tan^{-1}\left(\dfrac{x-1}{2}\right)\right)+3$ $\Rightarrow y=2\left(1+\tan^2\left(\tan^{-1}\left(\dfrac{x-1}{2}\right)\right)\right)+3$	M1	3.1a
	$\Rightarrow y=2\left(1+\left(\dfrac{x-1}{2}\right)^2\right)+3=\dfrac{1}{2}(x-1)^2+5$*	A1	2.1
	(b) Alternative 3:		
	$\dfrac{dy}{dx}=2\tan t=x-1\Rightarrow y=\int(x-1)dx=\dfrac{x^2}{2}-x+c$ $(3,7)\to 7=\dfrac{3^2}{2}-3+c\Rightarrow c=\dfrac{11}{2}$	M1	3.1a
	$\dfrac{x^2}{2}-x+\dfrac{11}{2}=\dfrac{1}{2}\left(x^2-2x\right)+\dfrac{11}{2}=\dfrac{1}{2}(x-1)^2-\dfrac{1}{2}+\dfrac{11}{2}=\dfrac{1}{2}(x-1)^2+5$*	A1	2.1

(c)	**Attempts the lower limit for *k*:** $\frac{1}{2}(x-1)^2+5=-\frac{1}{2}x+k \Rightarrow x^2-x+(11-2k)=0$ $b^2-4ac=1-4(11-2k)=0 \Rightarrow k=...$	M1	2.1
	$(k=)\frac{43}{8}$	A1	1.1b
	Attempts the upper limit for *k*: $(x,y)_{t=-\frac{\pi}{4}}: t=-\frac{\pi}{4} \Rightarrow x=2\tan\left(-\frac{\pi}{4}\right)+1=-1,\ y=2\sec^2\left(-\frac{\pi}{4}\right)+3=7$ $(-1,\ 7), y=-\frac{1}{2}x+k \Rightarrow 7=\frac{1}{2}+k \Rightarrow k=...$	M1	2.1
	$(k=)\frac{13}{2}$	A1	1.1b
	$\frac{43}{8}<k\leqslant\frac{13}{2}$	A1	2.2a
		(5)	
			(12 marks)
Notes:			

(a) **Must use parametric differentiation to score any marks in this part and not e.g. Cartesian form**

M1: For the key step of attempting $\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$. There must be some attempt to differentiate both parameters however poor and divide the right way round so using $\frac{dy}{dx}=\frac{y}{x}$ scores M0.

This may be implied by e.g. $\frac{dx}{dt}=2\sec^2 t,\ \frac{dy}{dt}=4\sec^2 t\tan t,\ t=\frac{\pi}{4} \Rightarrow \frac{dx}{dt}=4,\ \frac{dy}{dt}=8 \Rightarrow \frac{dy}{dx}=2$

A1: $\frac{dy}{dx}=\frac{4\sec^2 t\tan t}{2\sec^2 t}$. Correct expression in any form. May be implied as above.

Condone the confusion with variables as long as the intention is clear e.g.

$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{4\sec^2 x\tan x}{2\sec^2 x}(=2\tan x)$ and allow subsequent marks if this is interpreted correctly

M1: For attempting to find the values of *x*, *y* and the gradient at $t=\frac{\pi}{4}$ **AND** getting at least two correct.

Follow through on their $\frac{dy}{dx}$ so allow for any two of $x=3,\ y=7,\ \frac{dy}{dx}=2\left(\text{or their } \frac{dy}{dx} \text{ at } t=\frac{\pi}{4}\right)$

Note that the $x=3,\ y=7$ may be seen as e.g. (3, 7) on the diagram. There must be a non-trivial $\frac{dy}{dx}$ for this mark e.g. they must have a $\frac{dy}{dx}$ to substitute into.

M1: For a correct attempt at the normal equation using their *x* and *y* at $t=\frac{\pi}{4}$ with the negative reciprocal of their $\frac{dy}{dx}$ at $t=\frac{\pi}{4}$ having made some attempt at $\frac{dy}{dx}$ and all correctly placed.

For attempts using $y = mx + c$ they must reach as far as a value for *c* using their *x* and *y* at $t=\frac{\pi}{4}$ with the negative reciprocal of their $\frac{dy}{dx}$ at $t=\frac{\pi}{4}$ all correctly placed.

A1*: Proceeds with a clear argument to the given answer with no errors.

(b)

M1: Attempts to use $\sec^2 t = 1+\tan^2 t$ oe to obtain an equation involving *y* and $(x-1)^2$

E.g. as above or e.g. $y = 2\sec^2 t + 3 = 2\left(1+\tan^2 t\right)+3 = 2\left(1+\left(\frac{x-1}{2}\right)^2\right)+3$ for M1 and then

$y = \frac{1}{2}(x-1)^2 + 5*$ for A1

A1*: Proceeds with a clear argument to the given answer with no errors

Alternative 1:

M1: Uses the given result, substitutes for *x* and attempts to use $\sec^2 t = 1+\tan^2 t$ oe

A1: Proceeds with a clear argument to the *y* parameter and makes a (minimal) conclusion e.g. " = *y*" QED, hence proven etc.

Alternative 2:

M1: Uses the *x* parameter to obtain *t* in terms of arctan, substitutes into *y* and attempts to use $\sec^2 t = 1+\tan^2 t$ oe

A1: Proceeds with a clear argument to the given answer with no errors

Alternative 3:

M1: Uses $\frac{dy}{dx}$ from part (a) to express $\frac{dy}{dx}$ in terms of *x*, integrates and uses (3, 7) to find "*c*" to reach a Cartesian equation.

A1: Proceeds with a clear argument to the given answer with no errors

Allow the marks for (b) to score anywhere in their solution e.g. if they find the Cartesian equation in part (a)

(c)

M1: A full attempt to find the **lower** limit for *k*.

$\frac{1}{2}(x-1)^2 + 5 = -\frac{1}{2}x + k \Rightarrow x^2 - x + (11-2k) = 0 \Rightarrow b^2 - 4ac = 1 - 4(11-2k) = 0 \Rightarrow k = ...$

Score **M1** for setting $\frac{1}{2}(x-1)^2 + 5 = -\frac{1}{2}x + k$, rearranging to 3TQ form and attempts $b^2 - 4ac...0$ e.g. $b^2 - 4ac > 0$ or e.g. $b^2 - 4ac < 0$ correctly to find a value for *k*.

A1: $k = \frac{43}{8}$ oe. Look for this **value** e.g. may appear in an inequality e.g. $k > \frac{43}{8}$, $k < \frac{43}{8}$

An alternative method using calculus for lower limit:

$$y = \frac{1}{2}(x-1)^2 + 5 \Rightarrow \frac{dy}{dx} = x-1,\ x-1 = -\frac{1}{2} \Rightarrow x = \frac{1}{2}$$

$$x = \frac{1}{2} \Rightarrow y = \frac{1}{2}\left(\frac{1}{2}-1\right)^2 + 5 = \frac{41}{8}$$

$$y = -\frac{1}{2}x + k \Rightarrow \frac{41}{8} = -\frac{1}{4} + k \Rightarrow k = ...$$

Score **M1** for $\frac{dy}{dx}$ = "a linear expression in *x*", sets = $-\frac{1}{2}$, solves a linear equation to find *x* and then substitutes into the given result in (b) to find *y* and then uses $y = -\frac{1}{2}x + k$ to find a value for *k*. **A1**: $k = \frac{43}{8}$ oe. Look for this **value** e.g. may appear in an inequality e.g. $k > \frac{43}{8}$, $k < \frac{43}{8}$

An alternative method using parameters for lower limit:

$$y=-\frac{1}{2}x+k \Rightarrow 2\sec^2 t+3=-\frac{1}{2}(2\tan t+1)+k$$

$$\Rightarrow 2\left(1+\tan^2 t\right)+3=-\frac{1}{2}(2\tan t+1)+k \Rightarrow 2\tan^2 t+\tan t+5.5-k=0$$

$$b^2-4ac=0 \Rightarrow 1-4\times 2(5.5-k)=0 \Rightarrow k=\frac{43}{8}$$

Score **M1** for substituting parametric form of x and y into $y=-\frac{1}{2}x+k$, uses $\sec^2 t=1+\tan^2 t$ rearranges to 3TQ form and attempts $b^2-4ac...0$ or e.g. $b^2-4ac>0$ or $b^2-4ac<0$ correctly to find a value for k.

A1: $k=\frac{43}{8}$ oe. Look for this **value** e.g. may appear in an inequality e.g. $k>\frac{43}{8}$, $k<\frac{43}{8}$

M1: A full attempt to find the **upper** limit for k. This requires an attempt to find the value of x and the value of y using $t=-\frac{\pi}{4}$, the substitution of these values into $y=-\frac{1}{2}x+k$ and solves for k.

A1: $k=\frac{13}{2}$. Look for this value e.g. may appear in an inequality.

A1: Deduces the correct range for k: $\frac{43}{8}<k\leqslant\frac{13}{2}$

Allow equivalent notation e.g. $\left(k\leqslant\frac{13}{2} \text{ and } k>\frac{43}{8}\right)$, $\left(k\leqslant\frac{13}{2}\cap k>\frac{43}{8}\right)$, $\left(\frac{43}{8}, \frac{13}{2}\right]$

But not e.g. $\left(k\leqslant\frac{13}{2},\ k>\frac{43}{8}\right)$, $\left(k\leqslant\frac{13}{2}\cup k>\frac{43}{8}\right)$, $\left(k\leqslant\frac{13}{2} \text{ or } k>\frac{43}{8}\right)$ and do not allow if in terms of x.

Allow equivalent exact values for $\frac{43}{8}$, $\frac{13}{2}$

There may be other methods for finding the upper limit which are valid. If you are in any doubt if a method deserves credit then use Review.