3. A sequence of terms $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 3$$
$$a_{n+1} = 8 - a_n$$

(a) (i) Show that this sequence is periodic.

   (ii) State the order of this periodic sequence.

(2)

(b) Find the value of

$$\sum_{n=1}^{85} a_n$$

(2)

(a) (i) $a_1 = 3$

$a_2 = 8 - a_1 = 8 - 3 = 5$

$a_3 = 8 - a_2 = 8 - 5 = 3$ (1 mark)

a_3 is same as a_1 so sequence is periodic

(ii) sequence will continue
3 5 3 5 3 5 ...
with each term the same as that 2 before
so order of periodic sequence is 2   (1 mark)

(b) $\sum_{n=1}^{85} a_n = a_1 + a_2 + a_3 + a_4 + \ldots + a_{84} + a_{85}$

$= 3 + 5 + 3 + 5 + \ldots + 5 + 3$
$\left(a_i = 3 \text{ for odd } i \atop a_i = 5 \text{ for even } i \right)$

$= (3+5) + (3+5) + \ldots (3+5) + 3$

42 times because $\frac{84}{2} = 42$

$= 42(3+5) + 3$ (1 mark) $= 339$ (1 mark)