

5. The table below shows corresponding values of  $x$  and  $y$  for  $y = \log_3 2x$

The values of  $y$  are given to 2 decimal places as appropriate.

$x$	3	4.5	6	7.5	9
$y$	1.63	2	2.26	2.46	2.63

(a) Using the trapezium rule with all the values of  $y$  in the table, find an estimate for

$$\int_3^9 \log_3 2x \, dx$$

(3)

Using your answer to part (a) and making your method clear, estimate

(b) (i)  $\int_3^9 \log_3 (2x)^{10} \, dx$

(ii)  $\int_3^9 \log_3 18x \, dx$

(3)

$$\int_3^9 \log_3 2x \approx \frac{1}{2} h (1.63 + 2.63 + 2(2 + 2.26 + 2.46))$$

(1 mark)

$$h = \text{interval width} = \frac{\text{upper bound} - \text{lower bound}}{\text{no. of trapezia}}$$

$$= \frac{9-3}{4} = 1.5 \quad (1 \text{ mark})$$

so,

$$\int_3^9 \log_3 2x \approx \frac{1.5}{2} (1.63 + 2.63 + 2(2 + 2.26 + 2.46))$$

$$\approx 13.275 = 13.3 \text{ 1dp} \quad (1 \text{ mark})$$

(b)(i)  $\int_3^9 \log_3 (2x)^{10} = \int_3^9 10 \log_3 (2x) = 10 \int_3^9 \log_3 (2x)$

$$\approx 10 \times 13.275 = 132.75 \quad (1 \text{ mark})$$

(ii)  $\int_3^9 \log_3 18x = \int_3^9 \log_3 (9 \times 2x) = \int_3^9 \log_3 9 + \log_3 2x$

$$= \int_3^9 2 + \int_3^9 \log_3 2x = [2x]_3^9 + \int_3^9 \log_3 2x \approx 2(9) - 2(3) + 13.275$$

$$\approx 25.275 \quad (2 \text{ marks})$$