

6.

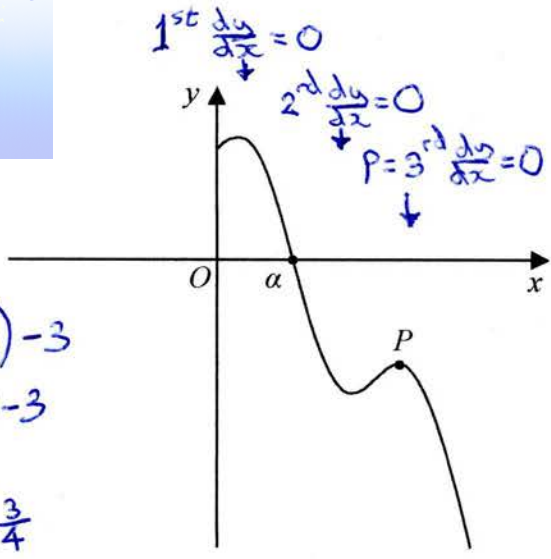


Figure 2

(a) $f'(x) = 8 \cos\left(\frac{1}{2}x\right)\left(\frac{1}{2}\right) - 3$
 $= 4 \cos\left(\frac{1}{2}x\right) - 3$
 (1 mark)

$f'(x) = 0 \Rightarrow \cos\left(\frac{1}{2}x\right) = \frac{3}{4}$
 $\Rightarrow \frac{1}{2}x = 0.7227\dots$
 (1 mark)

(a) contd.

$3^{rd} \frac{dy}{dx} = 0$
 $\frac{1}{2}x = 2\pi + 0.7227\dots$
 $x = 14.011\dots$
 $= 14.0 \text{ 3sf}$
 (1 mark)

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$ where

$$f(x) = 8 \sin\left(\frac{1}{2}x\right) - 3x + 9 \quad x > 0$$

and x is measured in radians.

The point P , shown in Figure 2, is a local maximum point on the curve.

Using calculus and the sketch in Figure 2,

(a) find the x coordinate of P , giving your answer to 3 significant figures.

(4)

The curve crosses the x -axis at $x = \alpha$, as shown in Figure 2.

Given that, to 3 decimal places, $f(4) = 4.274$ and $f(5) = -1.212$

(b) explain why α must lie in the interval $[4, 5]$ and f is continuous ie no asymptote (1 mark)

(b) there is a change of sign between $f(4)$ and $f(5)$

(c) Taking $x_0 = 5$ as a first approximation to α , apply the Newton-Raphson method once to $f(x)$ to obtain a second approximation to α .

Show your method and give your answer to 3 significant figures.

(2)

(c) $x_0 = 5$
 $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 5 - \frac{f(5)}{f'(5)}$
 $f(5) = 8 \sin\left(\frac{1}{2}(5)\right) - 3(5) + 9 = -1.212\dots$
 $f'(5) = 4 \cos\left(\frac{1}{2}(5)\right) - 3 = -6.204\dots$
 (1 mark)

$x_1 = 5 - \frac{-1.212\dots}{-6.204\dots} = 4.804\dots = 4.80 \text{ 3sf}$
 (1 mark)