

7. (a) Find the first four terms, in ascending powers of x , of the binomial expansion of

$$\sqrt{4-9x}$$

writing each term in simplest form.

(4)

A student uses this expansion with $x = \frac{1}{9}$ to find an approximation for $\sqrt{3}$

Using the answer to part (a) and without doing any calculations,

- (b) state whether this approximation will be an overestimate or an underestimate of $\sqrt{3}$ giving a brief reason for your answer.

(1)

$$\begin{aligned} \text{(a)} \quad (4-9x)^{\frac{1}{2}} &= \left(4\left(1-\frac{9}{4}x\right)\right)^{\frac{1}{2}} \\ &= 4^{\frac{1}{2}}\left(1-\frac{9}{4}x\right)^{\frac{1}{2}} \\ &= 2\left(1-\frac{9}{4}x\right)^{\frac{1}{2}} \end{aligned} \quad (1 \text{ mark})$$

$$\approx 2\left(1 + \frac{1}{2}\left(-\frac{9}{4}x\right) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(-\frac{9}{4}x\right)^2 + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}\left(-\frac{9}{4}x\right)^3 + \dots\right) \quad (2 \text{ marks})$$

$$\approx 2\left(1 - \frac{9}{8}x + \left(-\frac{1}{8}\right)\left(\frac{81}{16}x^2\right) + \left(\frac{3}{48}\right)\left(-\frac{729}{64}x^3\right) + \dots\right)$$

$$\approx 2\left(1 - \frac{9}{8}x - \frac{81}{128}x^2 - \frac{2187}{3072}x^3 - \dots\right)$$

$$\approx 2 - \frac{9}{4}x - \frac{81}{64}x^2 - \frac{729}{512}x^3 - \dots \quad (1 \text{ mark})$$

$$\text{(b)} \quad \text{with } x = \frac{1}{9}, \quad \sqrt{4-9x} = \sqrt{4-9\left(\frac{1}{9}\right)} = \sqrt{4-1} = \sqrt{3}$$

the approximation will be an overestimate because x is positive and every term after the first is a subtraction. More terms would bring us closer to true $\sqrt{3}$ by subtraction. Because we are not doing these subtractions, our approximation is an overestimate.

(1 mark)