

8.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

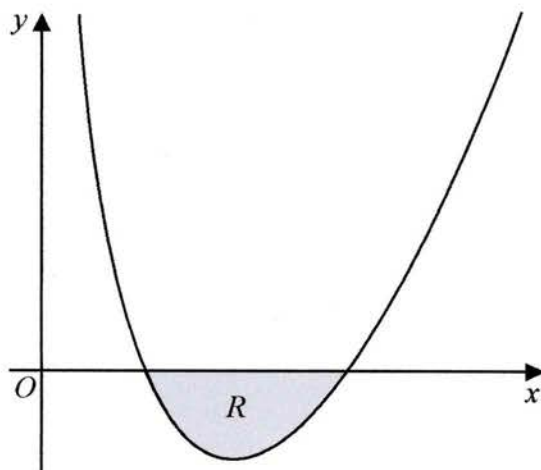


Figure 3

Figure 3 shows a sketch of part of a curve with equation

$$y = \frac{(x-2)(x-4)}{4\sqrt{x}} \quad x > 0$$

$y=0$ when
 $(x-2)=0 \Rightarrow x=2$
 $(x-4)=0 \Rightarrow x=4$
 so limits of integral
 are $x=2, 4$ (1 mark)

The region R , shown shaded in Figure 3, is bounded by the curve and the x -axis.Find the exact area of R , writing your answer in the form $a\sqrt{2} + b$, where a and b are constants to be found.

(6)

$$\begin{aligned}
 \text{Area, } R &= \int_2^4 y \, dx = \int_2^4 \frac{x^2 - 6x + 8}{4x^{\frac{1}{2}}} \, dx \\
 &\quad \text{because } R \text{ below } x\text{-axis} \\
 &= -\int_2^4 \frac{1}{4} x^{\frac{3}{2}} - \frac{3}{2} x^{\frac{1}{2}} + 2x^{-\frac{1}{2}} \, dx \quad (2 \text{ marks})
 \end{aligned}$$

$$\begin{aligned}
 &= -\left[\frac{2}{5} \left(\frac{1}{4} \right) x^{\frac{5}{2}} - \frac{3}{2} \left(\frac{2}{3} \right) x^{\frac{3}{2}} + \frac{2}{1} \left(2 \right) x^{\frac{1}{2}} \right]_2^4 \\
 &= -\left[\frac{1}{10} x^{\frac{5}{2}} - x^{\frac{3}{2}} + 4x^{\frac{1}{2}} \right]_2^4 \quad (2 \text{ marks}) \\
 &= -\left(\frac{1}{10} (4)^{\frac{5}{2}} - (4)^{\frac{3}{2}} + 4(4)^{\frac{1}{2}} \right) + \left(\frac{1}{10} (2)^{\frac{5}{2}} - (2)^{\frac{3}{2}} + 4(2)^{\frac{1}{2}} \right) \\
 &= -\frac{32}{10} + 8 - 8 + \frac{4\sqrt{2}}{10} - 2\sqrt{2} + 4\sqrt{2} = \frac{12}{5}\sqrt{2} - \frac{16}{5} \quad (1 \text{ mark})
 \end{aligned}$$