

11. Prove, using algebra, that

$$n(n^2 + 5)$$

is even for all $n \in \mathbb{N}$.

Either n is even or n is odd

$$n \text{ is even} \Rightarrow n = 2k \quad \text{for some integer } k$$

$$n \text{ is odd} \Rightarrow n = 2k + 1 \quad \text{for some integer } k \quad (1 \text{ mark})$$

$$\begin{aligned} n = 2k \Rightarrow n(n^2 + 5) &= 2k((2k)^2 + 5) \\ &= 2(k(2k^2 + 5)) \end{aligned}$$

\uparrow
which is even
because 2 is factor

(1 mark)

$$\begin{aligned} n = 2k + 1 \Rightarrow n(n^2 + 5) &= (2k + 1)((2k + 1)^2 + 5) \\ &= (2k + 1)(4k^2 + 4k + 1 + 5) \\ &= (2k + 1)(4k^2 + 4k + 6) \\ &= (2k + 1)2(2k^2 + 2k + 3) \end{aligned}$$

\uparrow
which is even
because 2 is factor

(1 mark)

$n(n^2 + 5)$ is even for even n and odd n
so $n(n^2 + 5)$ is even for all $n \in \mathbb{N}$ by exhaustion (1 mark)