



12. The function f is defined by

$$f(x) = \frac{e^{3x}}{4x^2 + k}$$

(a) Quotient Rule is
where $f = \frac{u}{v}$
 $f' = \frac{vu' - uv'}{v^2}$ (1 mark)

where k is a positive constant.

(a) Show that

$$f'(x) = (12x^2 - 8x + 3k)g(x)$$

where $g(x)$ is a function to be found.

(3)

Given that the curve with equation $y = f(x)$ has at least one stationary point,

(b) find the range of possible values of k .

(3)

(a) contd. $u = e^{3x} \Rightarrow u' = e^{3x} \times 3 = 3e^{3x}$
 $v = 4x^2 + k \Rightarrow v' = 8x$

$$f' = \frac{(4x^2 + k)(3e^{3x}) - (e^{3x})(8x)}{(4x^2 + k)^2} = \frac{12x^2 e^{3x} + 3ke^{3x} - 8xe^{3x}}{(4x^2 + k)^2}$$
$$= (12x^2 - 8x + 3k) \left(\frac{e^{3x}}{(4x^2 + k)^2} \right) \quad (2 \text{ marks})$$

(b) f has at least one stationary point
 $\Rightarrow f' = 0$ at least once

$f' = 0$ when numerator = 0
when $(12x^2 - 8x + 3k)(e^{3x}) = 0$

e^{3x} always > 0 , so $12x^2 - 8x + 3k = 0$ for stationary point (1 mark)

$12x^2 - 8x + 3k = 0$ when determinant ≥ 0
 $b^2 - 4ac \geq 0$
 $(-8)^2 - 4(12)(3k) \geq 0$ (1 mark)
 $64 - 144k \geq 0$
 $64 \geq 144k \div 16$
 $4 \geq 9k \div 9$
 $\frac{4}{9} \geq k$

Given $k > 0$, $0 < k \leq \frac{4}{9}$ (1 mark)