

13. Relative to a fixed origin  $O$

- the point  $A$  has position vector  $4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$
- the point  $B$  has position vector  $4\mathbf{j} + 6\mathbf{k}$
- the point  $C$  has position vector  $-16\mathbf{i} + p\mathbf{j} + 10\mathbf{k}$

where  $p$  is a constant.

Given that  $A, B$  and  $C$  lie on a straight line,

(a) find the value of  $p$ .

(3)

The line segment  $OB$  is extended to a point  $D$  so that  $\vec{CD}$  is parallel to  $\vec{OA}$

(b) Find  $|\vec{OD}|$ , writing your answer as a fully simplified surd.

(3)

(a)  $\vec{OA} = \begin{pmatrix} 4 \\ -3 \\ 5 \end{pmatrix}$   $\vec{OB} = \begin{pmatrix} 0 \\ 4 \\ 6 \end{pmatrix}$   $\vec{OC} = \begin{pmatrix} -16 \\ p \\ 10 \end{pmatrix}$

On straight line,  $\vec{AB}$  parallel to  $\vec{AC}$  (collinear because point in common)

$$\vec{AB} = \begin{pmatrix} 0-4 \\ 4-(-3) \\ 6-5 \end{pmatrix} = \begin{pmatrix} -4 \\ 7 \\ 1 \end{pmatrix} \quad \vec{AC} = \begin{pmatrix} -16-4 \\ p-(-3) \\ 10-5 \end{pmatrix} = \begin{pmatrix} -20 \\ p+3 \\ 5 \end{pmatrix} \quad (1 \text{ mark})$$

if parallel one vector is scalar multiple of the other

$$\Rightarrow \begin{pmatrix} -4 \\ 7 \\ 1 \end{pmatrix} = \lambda \begin{pmatrix} -20 \\ p+3 \\ 5 \end{pmatrix} \text{ for some } \lambda$$

$$-4 = -20\lambda \Rightarrow \lambda = \frac{1}{5} \quad \text{check, } 1 = \lambda 5 = \frac{1}{5}(5) \checkmark$$

$$\text{so, } 7 = \frac{1}{5}(p+3) \Rightarrow p+3 = 35 \Rightarrow p = 32 \quad (2 \text{ marks})$$

(b)  $\vec{OB}$  extended to  $D$ , so  $\vec{OD} = \mu \vec{OB}$  for some scalar  $\mu$

$$\vec{OD} = \begin{pmatrix} 0 \\ 4\mu \\ 6\mu \end{pmatrix} \quad \vec{CD} = \vec{OD} - \vec{OC} = \begin{pmatrix} 0-(-16) \\ 4\mu-p \\ 6\mu-10 \end{pmatrix} = \begin{pmatrix} 16 \\ 4\mu-32 \\ 6\mu-10 \end{pmatrix} \quad (1 \text{ mark})$$

$\vec{CD}$  parallel to  $\vec{OA} \Rightarrow \vec{CD} = k\vec{OA}$  for some scalar  $k$

$$\Rightarrow \begin{pmatrix} 16 \\ 4\mu-32 \\ 6\mu-10 \end{pmatrix} = \begin{pmatrix} 4k \\ -3k \\ 5k \end{pmatrix} \quad \begin{aligned} 16 &= 4k \Rightarrow k=4, \\ 4\mu-32 &= -3(4) = -12 \\ &\Rightarrow \mu=5 \end{aligned} \quad (1 \text{ mark})$$

$$\text{check, } 6(5)-10 = 20 = 5(4) \checkmark$$

$$\begin{aligned} \vec{OB} &= \begin{pmatrix} 0 \\ 4(5) \\ 6(5) \end{pmatrix} = \begin{pmatrix} 0 \\ 20 \\ 30 \end{pmatrix} \\ |\vec{OB}| &= \sqrt{0^2 + 20^2 + 30^2} \quad (1 \text{ mark}) \\ &= \sqrt{1300} = \sqrt{100 \times 13} = 10\sqrt{13} \end{aligned}$$