

15.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Given that the first three terms of a geometric series are

$12 \cos \theta$        $5 + 2 \sin \theta$       and       $6 \tan \theta$

(a) show that

$$4 \sin^2 \theta - 52 \sin \theta + 25 = 0$$

(c)  $S_{\infty} = \frac{a}{1-r}$

with  $\theta = \frac{5\pi}{6}$ ,

$a = 12 \cos\left(\frac{5\pi}{6}\right) = -6\sqrt{3}$  (1 mark)

(3)

Given that  $\theta$  is an obtuse angle measured in radians,

(e) cota  $r = \frac{5 + 2 \sin\left(\frac{5\pi}{6}\right)}{12 \cos\left(\frac{5\pi}{6}\right)} = -\frac{1}{\sqrt{3}}$  (1 mark)

(2)

(b) solve the equation in part (a) to find the exact value of  $\theta$

(c) show that the sum to infinity of the series can be expressed in the form

(c) cota  $S_{\infty} = \frac{-6\sqrt{3}}{1 - \left(-\frac{1}{\sqrt{3}}\right)} = \frac{-6\sqrt{3}}{1 + \frac{1}{\sqrt{3}}}$  (1 mark)

where  $k$  is a constant to be found.

(c) cota  $S_{\infty} = \frac{k(1-\sqrt{3})}{\frac{\sqrt{3}+1}{\sqrt{3}}} = \frac{-6\sqrt{3}\sqrt{3}}{\sqrt{3}+1} = \frac{-18+18\sqrt{3}}{1+\sqrt{3}}$  (1 mark) (5)

(a) Because geometric series,

$r = \frac{5 + 2 \sin \theta}{12 \cos \theta} = \frac{6 \tan \theta}{5 + 2 \sin \theta}$  (1 mark)

$(5 + 2 \sin \theta)^2 = (6 \tan \theta)(12 \cos \theta)$

$25 + 20 \sin \theta + 4 \sin^2 \theta = 72 \left(\frac{\sin \theta}{\cos \theta}\right) \cos \theta$  (1 mark)

$25 + 20 \sin \theta + 4 \sin^2 \theta = 72 \sin \theta$

$\Rightarrow 4 \sin^2 \theta - 52 \sin \theta + 25 = 0$  (1 mark)

(c) cota  $S_{\infty} = \frac{-18+18\sqrt{3}}{-2} = 9-9\sqrt{3} = 9(1-\sqrt{3})$  (1 mark)

(b)  $4 \sin^2 \theta - 2 \sin \theta - 50 \sin \theta + 25 = 0$

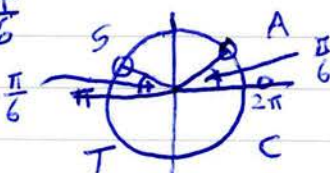
$2 \sin \theta (2 \sin \theta - 1) - 25 (2 \sin \theta - 1) = 0$

$(2 \sin \theta - 25)(2 \sin \theta - 1) = 0$

$\sin \theta = \frac{25}{2}, \frac{1}{2}$  (1 mark)

not possible for sin

$\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$



Given  $\theta$  is obtuse,

$\theta$  in 2<sup>nd</sup> quadrant

$\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$  (1 mark)