



(a) contd. At $t = \frac{\pi}{4}$,

$$\frac{dy}{dx} = 2 \tan\left(\frac{\pi}{4}\right) = 2(1) = 2$$

$$x = 2 \tan\left(\frac{\pi}{4}\right) + 1 = 2(1) + 1 = 3$$

$$y = 2 \sec^2\left(\frac{\pi}{4}\right) + 3 = \frac{2}{\cos^2\left(\frac{\pi}{4}\right)} + 3 = \frac{2}{\left(\frac{1}{\sqrt{2}}\right)^2} + 3 = \frac{2}{\left(\frac{1}{2}\right)} + 3 = 4 + 3 = 7 \text{ (1 mark)}$$

16.

(a) $\frac{dx}{dt} = 2 \sec^2 t$

$$\frac{dy}{dt} = 4 \sec t \times \sec t \tan t = 4 \sec^2 t \tan t$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4 \sec^2 t \tan t}{2 \sec^2 t} = 2 \tan t \text{ (2 marks)}$$

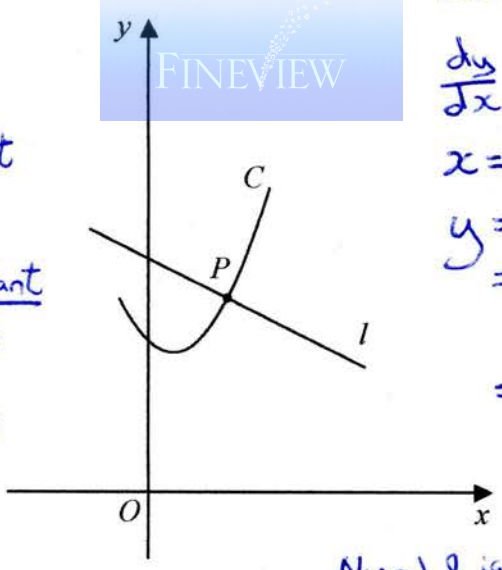


Figure 6

Normal, l is $y = -\frac{1}{2}x + c$
 At $(3, 7)$, $7 = -\frac{1}{2}(3) + c \Rightarrow c = \frac{17}{2}$
 So, l is $y = -\frac{1}{2}x + \frac{17}{2}$ (2 marks)

Figure 6 shows a sketch of the curve C with parametric equations

$$x = 2 \tan t + 1 \quad y = 2 \sec^2 t + 3 \quad -\frac{\pi}{4} \leq t \leq \frac{\pi}{3}$$

The line l is the normal to C at the point P where $t = \frac{\pi}{4}$

(a) Using parametric differentiation, show that an equation for l is

(b) RHS = $\frac{1}{2}(2 \tan t + 1)^2 + 5$
 $= 2 \tan^2 t + 5$
 $= 2(\sec^2 t - 1) + 5$
 $= 2 \sec^2 t - 2 + 5 = 2 \sec^2 t + 3$
 $= \text{LHS} \text{ (2 marks)}$
 (5)

(c) contd. upper limit is when x is most negative
 $2 \tan t + 1$ is most negative
 $\Rightarrow t = -\frac{\pi}{4}$ ($x = -1$, $y = 7$) (1 mark)

$$y = -\frac{1}{2}x + \frac{17}{2}$$

(b) Show that all points on C satisfy the equation

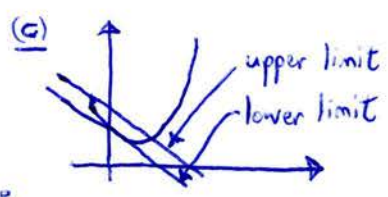
(c) contd. At $(-1, 7)$, line is
 $7 = -\frac{1}{2}(-1) + k$
 $\Rightarrow k = \frac{13}{2}$ (1 mark)

$$y = \frac{1}{2}(x-1)^2 + 5$$

(2)

The straight line with equation

so, $\frac{43}{8} < k \leq \frac{13}{2}$ (1 mark) $y = -\frac{1}{2}x + k$
 where k is a constant
 touch at $k = \frac{43}{8}$ is not 2 distinct intersects C at two distinct points.



(5)

(c) Find the range of possible values for k .

(e) contd. lower limit is where curve touches line
 curve = line has one repeated soln.

$$\frac{1}{2}(x-1)^2 + 5 = -\frac{1}{2}x + k$$

$$\frac{1}{2}(x^2 - 2x + 1) + 5 = -\frac{1}{2}x + k$$

$$x^2 - 2x + 1 + 10 = -x + 2k$$

$$x^2 - x + (11 - 2k) = 0$$

discriminant, $b^2 - 4ac = 0$

$$(-1)^2 - 4(1)(11 - 2k) = 0 \Rightarrow k = \frac{43}{8} \text{ (2 marks)}$$