

Question	Scheme	Marks	AOs
<b>15(a)</b>	$A = 1$	B1	1.1b
	$\frac{u^2}{u^2-1} \equiv A + \frac{B}{u+1} + \frac{C}{u-1}$ $\Rightarrow u^2 \equiv A(u^2-1) + B(u-1) + C(u+1) \Rightarrow B = \dots, C = \dots$	M1	1.1b
	$B = -\frac{1}{2}, C = \frac{1}{2}$	A1	1.1b
	<b>(3)</b>		
<b>(b)</b>	$u = \sqrt{1+e^{3x}} \Rightarrow u^2 = 1+e^{3x} \Rightarrow 2u \frac{du}{dx} = 3e^{3x}$ <b>or</b> $u = \sqrt{1+e^{3x}} \Rightarrow \frac{du}{dx} = \frac{3}{2}e^{3x}(1+e^{3x})^{-\frac{1}{2}}$	B1	1.1b
	$x = \frac{1}{3} \ln 3 \Rightarrow u = \sqrt{1+e^{\ln 3}} = \dots$ $x = \frac{1}{3} \ln 8 \Rightarrow u = \sqrt{1+e^{\ln 8}} = \dots$	M1	1.1b
	$\int \sqrt{1+e^{3x}} dx = \int u \times \frac{2u}{3e^{3x}} du = \int u \times \frac{2u}{3(u^2-1)} du$ <b>or</b> $\int \sqrt{1+e^{3x}} dx = \int \sqrt{1+e^{3x}} \times \frac{2}{3} \frac{\sqrt{1+e^{3x}}}{e^{3x}} dx = \int \frac{2u^2}{3(u^2-1)} du$	M1 A1	3.1a 1.1b
	$\int_{\frac{1}{3} \ln 3}^{\frac{1}{3} \ln 8} \sqrt{1+e^{3x}} dx = \frac{2}{3} \int_2^3 \frac{u^2}{u^2-1} du$	A1	2.1
	<b>(5)</b>		
<b>(c)</b>	$\int \frac{u^2}{u^2-1} du = \int \left( 1 - \frac{1}{2(u+1)} + \frac{1}{2(u-1)} \right) du$ $= u - \frac{1}{2} \ln(u+1) + \frac{1}{2} \ln(u-1) (+c)$	M1 A1	2.1 1.1b
	$\frac{2}{3} \int_2^3 \frac{u^2}{u^2-1} du = \frac{2}{3} \left[ u - \frac{1}{2} \ln(u+1) + \frac{1}{2} \ln(u-1) \right]_2^3$ $\frac{2}{3} \left( 3 - \frac{1}{2} \ln(4) + \frac{1}{2} \ln(2) - \left( 2 - \frac{1}{2} \ln(3) + \frac{1}{2} \ln(1) \right) \right) = \dots$	M1	2.1
	$= \frac{2}{3} + \frac{1}{3} \ln \frac{3}{2}$	A1	1.1b
	<b>(4)</b>		
	<b>(12 marks)</b>		

## Notes

(a)

B1: Correct value for  $A$

M1: Complete method to find  $B$  and  $C$  e.g. substitutes values or compares coefficients.

A1: Correct values (or fractions)

(b)

B1: Any correct equation connecting  $du$  with  $dx$

M1: Attempts to find limits in terms of  $u$  using the given substitution

M1: A completely correct strategy to achieve an integral in terms of  $u$  only

A1: Obtains  $\frac{2}{3} \int \frac{u^2}{u^2-1} du$  with no errors ignoring limits

A1: All correct with no errors and with correct limits

(c)

M1: Recognises the form for the integration for at least one of the fractions e.g.

$$\int \frac{A}{u+1} du \rightarrow k \ln(u+1) \quad \text{or} \quad \int \frac{B}{u-1} du \rightarrow k \ln(u-1)$$

A1: Fully correct integration

M1: Substitutes both  $u$  limits and subtracts and reaches an expression of the required form

A1: Correct answer