

6. The curve C has equation

$$y = ax^3 + bx^2 + 12x + 2$$

where a and b are constants.

(a) Find, in terms of a and b ,

$$(i) \frac{dy}{dx} \quad \underline{(a)(i)} \quad \frac{dy}{dx} = 3ax^{3-1} + 2bx^{2-1} + 12 \\ = 3ax^2 + 2bx + 12 \quad (2 \text{ marks})$$

$$(ii) \frac{d^2y}{dx^2} \quad \underline{(a)(ii)} \quad \frac{d^2y}{dx^2} = 2(3)ax^{2-1} + 2b \\ = 6ax + 2b \quad (1 \text{ mark}) \quad (3)$$

Given that

- the point $P\left(\frac{3}{2}, \frac{13}{2}\right)$ lies on C
 - $\frac{d^2y}{dx^2} = 0$ at P
- (c) At P , $\frac{d^2y}{dx^2} = 0$, so P could be a point of inflection.
 Just to left of P , $\frac{d^2y}{dx^2}(1.4) = 6(2)(1.4) + 2(-9) = -1.2 < 0$
 Just to right of P , $\frac{d^2y}{dx^2}(1.6) = 6(2)(1.6) + 2(-9) = 1.2 > 0$ (1 mark)

(b) find the value of a and the value of b .

(c) Show that P is a point of inflection.

$$\begin{array}{l} \swarrow \frac{d^2y}{dx^2} > 0 \text{ change of sign} \\ \searrow \frac{d^2y}{dx^2} < 0 \text{ in } \frac{d^2y}{dx^2} \end{array} \quad (2)$$

\Rightarrow point of inflection at P (1 mark) (2)

(b) At P ,

$$\frac{13}{2} = a\left(\frac{3}{2}\right)^3 + b\left(\frac{3}{2}\right)^2 + 12\left(\frac{3}{2}\right) + 2$$

$$\frac{13}{2} = \frac{27a}{8} + \frac{9b}{4} + 18 + 2$$

$$\frac{27a}{8} + \frac{9b}{4} = \frac{13}{2} - 18 - 2 = -\frac{27}{2}$$

$$27a + 18b = -108$$

$$3a + 2b = -12$$

At P ,

$$0 = 6a\left(\frac{3}{2}\right) + 2b \Rightarrow 9a + 2b = 0$$

Solving simultaneously,

$$9a + 2b = 0$$

$$-(3a + 2b = -12)$$

$$6a = 12$$

$$\Rightarrow \underline{a = 2} \quad \& \quad 9(2) + 2b = 0 \Rightarrow \underline{b = -9} \quad (2 \text{ marks})$$