



12.

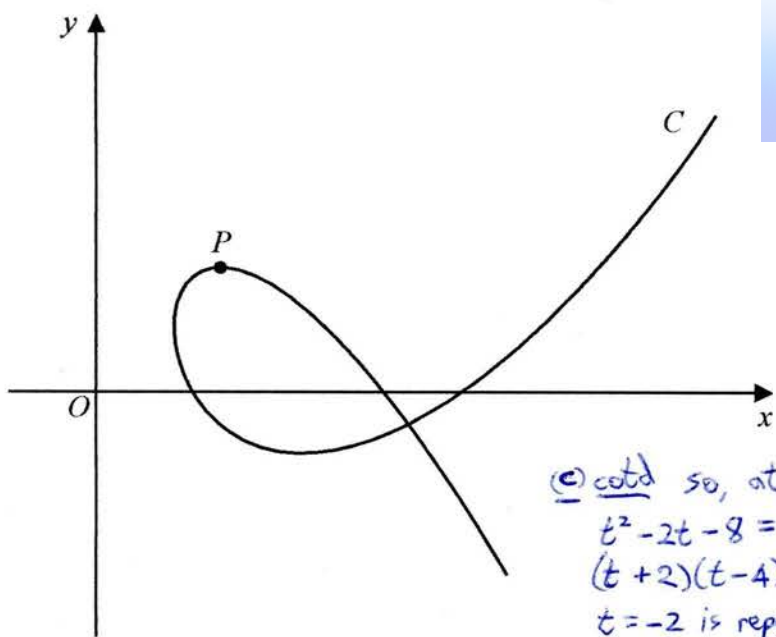


Figure 3

(c) cotd so, at Q
 $t^2 - 2t - 8 = 0$
 $(t+2)(t-4) = 0$
 $t = -2$ is repeated root at P,
 so $t = 4$ at Q, $y = 21$ (1 mark)
 $x = 4^2 + 4 + 4 = 24$
 Q is (24, 21) (1 mark)

Figure 3 shows a sketch of the curve C with parametric equations

$$x = t^2 + t + 4 \quad y = t^3 - 12t + 5 \quad -5 \leq t \leq 5$$

(a) Find $\frac{dy}{dx}$ in terms of t .

(a) $\frac{dx}{dt} = 2t + 1$ $\frac{dy}{dt} = 3t^2 - 12$
 $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2 - 12}{2t + 1}$ (2 marks) (2)

Given that the point P lies on C such that the tangent to C at P is parallel to the x-axis,

(b) find the coordinates of P.

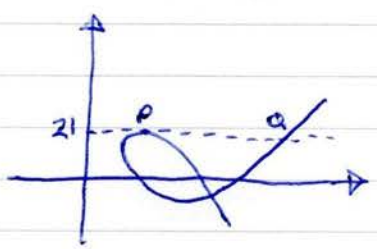
(b) At P, $\frac{dy}{dx} = 0 \Rightarrow$ numerator = 0
 $\Rightarrow 3t^2 - 12 = 0 \Rightarrow t = \pm 2$ (2)

Given that this tangent meets C again at the point Q, $t = +2 \Rightarrow x = 4 + 2 + 4 = 10, y = 8 - 24 + 5 = -11$

(c) find, using algebra, the coordinates of Q.

$t = -2 \Rightarrow x = 4 - 2 + 4 = 6, y = -8 + 24 + 5 = 21$
 From sketch, x & y are both positive at P, so (3)
 $t = -2$ and P is (6, 21) (2 marks)

(c) At Q, $y = 21$



$21 = t^3 - 12t + 5 \Rightarrow t^3 - 12t - 16 = 0$ (1 mark)

we know $t = -2$ is a solution

$$\begin{array}{r} t^2 - 2t - 8 \\ t+2 \overline{) t^3 + 0t^2 - 12t - 16} \\ \underline{-(t^3 + 2t^2)} \\ -2t^2 - 12t \\ \underline{-(-2t^2 - 4t)} \\ -8t - 16 \\ \underline{-(-8t - 16)} \\ 0 \end{array}$$