



14. The curve C has equation

Quotient Rule: $y = \frac{u}{v}$

$$y' = \frac{vu' - uv'}{v^2}$$

$$y = \frac{e^{2x}}{x^3 + 4} \quad x > -1$$

(a) Find $\frac{dy}{dx}$ (a) $u = e^{2x}$ $u' = 2e^{2x}$
 $v = x^3 + 4$ $v' = 3x^2$

$$y' = \frac{(x^3 + 4)(2e^{2x}) - (e^{2x})(3x^2)}{(x^3 + 4)^2} \quad (3 \text{ marks}) \quad (3)$$

The point P with x coordinate p lies on C.
 The line l is the tangent to C at P.

Given that l passes through the origin,

(b) show that $x = p$ is a solution of the equation

(b) At P, $y = \frac{e^{2p}}{p^3 + 4}$

So, P is $(p, \frac{e^{2p}}{p^3 + 4})$

Gradient at P, $y'(p)$

$$= \frac{(p^3 + 4)2e^{2p} - e^{2p}(3p^2)}{(p^3 + 4)^2} \quad (1 \text{ mark}) = \frac{e^{2p}(2p^3 - 3p^2 + 8)}{(p^3 + 4)^2} \quad (3)$$

The iteration formula

$$x^4 - 2x^3 + 4x - 2 = 0$$

tangent l goes through origin
 so l is $\frac{y}{x} = m$

$$x_{n+1} = \frac{2x_n^3 + 2}{x_n^3 + 4}$$

$(\frac{y}{x} = m)$

$$\frac{e^{2p}}{p^3 + 4} \left(\frac{1}{p}\right) = \frac{e^{2p}(2p^3 - 3p^2 + 8)}{(p^3 + 4)^2} \quad (1 \text{ mark})$$

with $x_1 = 0.5$ is used to find an approximation for p.

$$\Rightarrow p^3 + 4 = 2p^4 - 3p^3 + 8p$$

$$\Rightarrow 2p^4 - 4p^3 + 8p - 4 = 0$$

(c) Use the iteration formula to find the value, to 4 decimal places, of

(i) x_2

$$\Rightarrow p^4 - 2p^3 + 4p - 2 = 0 \quad (1 \text{ mark})$$

(ii) p

(d) Hence find the gradient of l, giving your answer to 2 decimal places.

(c)(i) $x_2 = \frac{2x_1^3 + 2}{x_1^3 + 4} = \frac{2(0.5)^3 + 2}{(0.5)^3 + 4} = \frac{6}{11} = 0.54545 \dots$ (2 marks)

- (ii) $x_3 = 0.558483 \dots$
- $x_4 = 0.562596 \dots$
- $x_5 = 0.563930 \dots$
- $x_6 = 0.564366 \dots$
- $x_7 = 0.564509 \dots$
- $x_8 = 0.564556 \dots$
- $x_9 = 0.564571 \dots$

$$x = \frac{2 \langle \text{Ans} \rangle^3 + 2}{\langle \text{Ans} \rangle^3 + 4}$$

x_8 & x_9 agree to 4 dp
 so $p = 0.5646$ 4dp (1 mark)

(d) Gradient = $\frac{y_p}{x_p} = \frac{e^{2p}}{p^3 + 4} \left(\frac{1}{p}\right) = \frac{e^{2(0.5646)}}{(0.5646)^3 + 4} \left(\frac{1}{0.5646}\right) = 1.310 \dots$
 $= 1.31$ 2dp (1 mark)