

15. (a) Find the values of the constants A , B and C for which

(a) start by dividing numerator by denominator for whole number A .

$$\frac{u^2}{u^2 - 1} \equiv A + \frac{B}{u + 1} + \frac{C}{u - 1}$$

(3)

(b) Use the substitution $u = \sqrt{1 + e^{3x}}$ to show that

$$u^2 - 1 \frac{1}{u^2 + 0} - \frac{1}{(u^2 - 1)}$$

$$\Rightarrow 1 + \frac{1}{u^2 - 1}$$

$\Rightarrow A = 1$ (1 mark)

$$\int_{\frac{1}{3} \ln 3}^{\frac{1}{3} \ln 8} \sqrt{1 + e^{3x}} dx = k \int_{\alpha}^{\beta} \frac{u^2}{u^2 - 1} du$$

(a) contd

$$\frac{1}{u^2 - 1} \equiv \frac{1}{(u+1)(u-1)} \equiv \frac{B}{u+1} + \frac{C}{u-1}$$

$$1 \equiv B(u-1) + C(u+1)$$

$$\text{When } u=1, 1 = 2C \Rightarrow C = \frac{1}{2}$$

$$\text{When } u=-1, 1 = -2B \Rightarrow B = -\frac{1}{2}$$

(2 marks)

(5)

where k , α and β are constants to be found.

(c) Using the answers from parts (a) and (b) and algebraic integration, show that

(b) $u = (1 + e^{3x})^{\frac{1}{2}}$

$$\frac{du}{dx} = \frac{1}{2} (1 + e^{3x})^{-\frac{1}{2}} (e^{3x}) 3$$

$$= \frac{3}{2} e^{3x} (1 + e^{3x})^{-\frac{1}{2}} \text{ (1 mark)}$$

$$\int_{\frac{1}{3} \ln 3}^{\frac{1}{3} \ln 8} \sqrt{1 + e^{3x}} dx = p + q \ln r$$

(b) contd

$$(1 + e^{3x})^{-\frac{1}{2}} = \frac{1}{u}$$

$$e^{3x} = u^2 - 1$$

$$\text{So, } \frac{du}{dx} = \frac{3}{2} (u^2 - 1) \left(\frac{1}{u}\right)$$

$$dx = \frac{2}{3} \frac{u}{u^2 - 1} du$$

(4)

where p , q and r are rational numbers to be found.

(b) contd $\int_{x = \frac{1}{3} \ln 3}^{x = \frac{1}{3} \ln 8} (1 + e^{3x})^{\frac{1}{2}} dx$

$$= \int_{u=2}^{u=3} \frac{2}{3} \frac{u}{u^2 - 1} du$$

Limits: x	$u = (1 + e^{3x})^{\frac{1}{2}}$
$\frac{1}{3} \ln 3$	$(1 + 3)^{\frac{1}{2}} = 2$
$\frac{1}{3} \ln 8$	$(1 + 8)^{\frac{1}{2}} = 3$

(1 mark)

$$= \frac{2}{3} \int_2^3 \frac{u}{u^2 - 1} du \quad (3 \text{ marks})$$

(c) from (a), Integral = $\frac{2}{3} \int_2^3 \left(1 - \frac{1}{2} \left(\frac{1}{u+1}\right) + \frac{1}{2} \left(\frac{1}{u-1}\right) \right) du$

$$= \frac{2}{3} \left[u - \frac{1}{2} \ln(u+1) + \frac{1}{2} \ln(u-1) \right]_2^3 \quad (2 \text{ marks})$$

$$= \frac{2}{3} \left(\left(3 - \frac{1}{2} \ln(4) + \frac{1}{2} \ln(2) \right) - \left(2 - \frac{1}{2} \ln(3) + \frac{1}{2} \ln(1) \right) \right) \text{ (1 mark)}$$

$$= \frac{2}{3} \left(3 - \frac{1}{2} \ln(4) + \frac{1}{2} \ln(2) - 2 + \frac{1}{2} \ln(3) - \frac{1}{2} \ln(1) \right)$$

$$= \frac{2}{3} \left(1 + \frac{1}{2} \ln \left(\frac{2 \times 3}{4 \times 1} \right) \right) = \frac{2}{3} \left(1 + \frac{1}{2} \ln \left(\frac{3}{2} \right) \right)$$

$$= \frac{2}{3} + \frac{1}{3} \ln \left(\frac{3}{2} \right) \text{ (1 mark)}$$