

| Question | Scheme | Marks | AOs |
|--|--|--------------|------|
| 9(a) Way 1 | $x = (t + 3)^2 - 25$ | M1 | 1.1b |
| | $\Rightarrow x + 25 = (t + 3)^2 \Rightarrow (x + 25)^{\frac{1}{2}} = (t + 3) \Rightarrow y = \dots$ | M1 | 2.1 |
| | $y = 6 \ln(x + 25)^{\frac{1}{2}} \Rightarrow y = 3 \ln(x + 25)$ | A1cso | 1.1b |
| | | (3) | |
| (a) Way 2 | | | |
| | $y = 6 \ln(t + 3) = 3 \ln(t + 3)^2$ | M1 | 1.1b |
| | $y = 3 \ln(t + 3)^2 = 3 \ln(t^2 + 6t + 9) = 3 \ln(x + 16 + 9)$ | M1 | 2.1 |
| | $y = 3 \ln(x + 25)$ | A1cso | 1.1b |
| (a) Way 3 | | | |
| | $y = 6 \ln(t + 3) \Rightarrow \frac{y}{6} = \ln(t + 3) \Rightarrow t + 3 = e^{\frac{y}{6}} \Rightarrow t = e^{\frac{y}{6}} - 3$ | M1 | 1.1b |
| | $x = \left(e^{\frac{y}{6}} - 3\right)^2 + 6\left(e^{\frac{y}{6}} - 3\right) - 16 \Rightarrow y = \dots$ or $x = \left(e^{\frac{y}{6}} - 3 + 8\right)\left(e^{\frac{y}{6}} - 3 - 2\right) \Rightarrow y = \dots$ | M1 | 2.1 |
| | $y = 3 \ln(x + 25)$ | A1cso | 1.1b |
| (a) Way 4 | | | |
| | $x = (t + 3)^2 - 25$ | M1 | 1.1b |
| | $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\left(\frac{6}{t+3}\right)}{2t+6} \Rightarrow \frac{3}{(t+3)^2} = \frac{3}{x+25} \Rightarrow y = 3 \ln(x+25) (+c)$ | M1 | 2.1 |
| | e.g. $t = 0 \Rightarrow x = -16, y = 6 \ln 3 \Rightarrow 6 \ln 3 = 3 \ln(9) \Rightarrow c = 0$ $y = 3 \ln(x + 25)$ | A1cso | 1.1b |
| (b) | $x = 0, y = 3 \ln 25$ oe e.g. $6 \ln 5$ | B1ft | 2.2a |
| | $\frac{dy}{dx} = \frac{3}{x + "25"} \Rightarrow \frac{dy}{dx} = \frac{3}{0 + "25"} \left(= \frac{3}{25} \right)$ or $\frac{dy}{dx} = \frac{\left(\frac{6}{t+3}\right)}{2t+6} \Rightarrow \frac{6}{2 \times 2 + 6} \left(= \frac{6}{50} = \frac{3}{25} \right)$ | M1 | 2.1 |
| | $y - "3 \ln 25" = \frac{3}{25} (x - 0)$ | dM1 | 3.1a |
| | $25y - 3x = 150 \ln 5$ | A1 | 2.2a |
| | | (4) | |
| (7 marks) | | | |
| Notes | | | |
| Choose the mark scheme that best matches their chosen method. | | | |

(a)

Way 1

M1: Attempts to complete the square. Award for sight of $x = (t + 3)^2 \pm \dots$ where $\dots \neq 0$

M1: Rearranges their $x = (t + 3)^2 - 25$ to either $(t + 3) = \dots$ or $(t + 3)^2 = \dots$ and then substitutes correctly their expression into the parametric equation for y . So e.g., $t = \sqrt{x + 25} - 3 \rightarrow y = 6 \ln(\sqrt{x + 25} - 3)$ is M0.

A1cso: $y = 3 \ln(x + 25)$ including brackets with all stages of working shown.

The “y =” must appear at some point.

Way 2

M1: Attempts to use the power rule for logarithms $y = 6 \ln(t + 3) = \dots \ln(t + 3)^2$ where $\dots \neq 6$

M1: Writes $y = 6 \ln(t + 3)$ as $3 \ln(t + 3)^2$ and then multiplies out and substitutes correctly in for t to obtain a Cartesian equation for C

A1cso: $y = 3 \ln(x + 25)$ including brackets with all stages of working shown.

The “y =” must appear at some point.

Way 3

M1: Attempts to make t the subject for $y = 6 \ln(t + 3)$ to obtain $t = e^{\frac{y}{6}} \pm \dots$ where $\dots \neq 0$

M1: Substitutes $t = e^{\frac{y}{6}} \pm \dots$ correctly into $x = t^2 + 6t - 16$ and rearranges to make y the subject.

A1cso: $y = 3 \ln(x + 25)$ including brackets with all stages of working shown.

The “y =” must appear at some point.

Way 4

M1: Attempts to complete the square. Award for sight of $x = (t + 3)^2 \pm \dots$ where $\dots \neq 0$

M1: Attempts to find $\frac{dy}{dx}$ where $\frac{dy}{dx} = \frac{\left(\frac{\dots}{t+3}\right)}{at+b}$, $a, b \neq 0$ and uses the completed square form to find $\frac{dy}{dx}$ in terms of x and then integrates to obtain a Cartesian equation for C

A1cso: A complete method using any correct point on the curve to show that $c = 0$ and obtain $y = 3 \ln(x + 25)$ with all stages of working shown. The “y =” must appear at some point.

Note that a common incorrect approach in (a) is:

$$x = t^2 + 6t - 16 = (t - 2)(t + 8) \Rightarrow x = t - 2 \Rightarrow t = x + 2 \Rightarrow y = 6 \ln(x + 5)$$

which scores no marks.

(b)

B1ft: Deduces $y = 3\ln 25$ or e.g. $y = 6\ln 5$ but allow follow through on their Cartesian equation with $x = 0$ and apply isw after a correct value or ft value for y

M1: Attempts to find $\frac{dy}{dx}$ when $x = 0$ so score for obtaining $\frac{dy}{dx} = \frac{\dots}{x + "25"}$ and substituting in $x = 0$

Allow this mark if they use the letters A and B e.g. $\frac{dy}{dx} = \frac{\dots}{x + B} = \frac{\dots}{0 + B}$ or allow a "made up" A and B .

or

Attempts to find $\frac{dy}{dx}$ when $t = 2$ by finding $\frac{dy}{dx} = \left(\frac{6}{t+3}\right) \Rightarrow \frac{6}{2 \times 2 + 6} \left(= \frac{6}{50} = \frac{3}{25} \right)$

For the derivative look for $\frac{dy}{dx} = \frac{\left(\frac{\dots}{t+3}\right)}{at+b}$ or e.g. $\left(\frac{\dots}{t+3}\right) \times \frac{1}{at+b}$ $a, b \neq 0$

NOTE if candidates find $\frac{dy}{dx} = \frac{\left(\frac{6}{t+3}\right)}{2t+6} = \frac{6(2t+6)}{t+3} = 12$ we will give BOD that $t = 2$ has been used unless

there is clear evidence that $t = 2$ has not been used.

dM1: Attempts to find the equation of the tangent. Score for sight of $y - "3\ln 25" = \frac{3}{25}(x\{-0\})$ or if they use

$y = mx + c$ they must proceed as far as $c = \dots$ **It is dependent on the previous method mark.**

Must have numeric A and B now.

A1: $25y - 3x = 150\ln 5$ or any integer multiple of this equation in the form $ax + by = c \ln 5$