

2. A sequence  $u_1, u_2, u_3, \dots$  is defined by

$$u_1 = 35$$

$$u_{n+1} = u_n + 7 \cos\left(\frac{n\pi}{2}\right) - 5(-1)^n$$

(a) (i) Show that  $u_2 = 40$

(ii) Find the value of  $u_3$  and the value of  $u_4$

(3)

Given that the sequence is periodic with order 4

(b) (i) write down the value of  $u_5$

(ii) find the value of  $\sum_{r=1}^{25} u_r$

(3)

$$\underline{\text{(a) (i)}} \quad u_2 = u_1 + 7 \cos\left(\frac{1\pi}{2}\right) - 5(-1)^1$$

$$= 35 + 7 \cos\left(\frac{\pi}{2}\right) + 5$$

$$= 40 + 7(0) = 40$$

$$\underline{\text{(ii)}} \quad u_3 = 40 + 7 \cos\left(\frac{2\pi}{2}\right) - 5(-1)^2 = 40 + 7(-1) - 5 = 28$$

$$u_4 = 28 + 7 \cos\left(\frac{3\pi}{2}\right) - 5(-1)^3 = 28 + 7(0) + 5 = 33$$

$$\underline{\text{(b) (i)}} \quad \begin{array}{cccccccc} 35 & 40 & 28 & 33 & 35 & 40 & 28 & 33 & \Rightarrow u_5 = 35 \\ u_1 & u_2 & u_3 & u_4 & u_5 & u_6 & u_7 & u_8 & \\ & & & & u_1 & u_2 & u_3 & u_4 & \end{array}$$

$$\underline{\text{(ii)}} \quad \underbrace{35 + 40 + 28 + 33}_{136} + \underbrace{35 + 40 + 28 + 33}_{136} + \dots$$

$$\frac{25}{4} = 6 \text{ r. } 1$$

$$(6 \times 136) + 35 = 851$$