

4. Coffee is poured into a cup.

The temperature of the coffee, H °C, t minutes after being poured into the cup is modelled by the equation

$$H = Ae^{-Bt} + 30$$

where A and B are constants.

Initially, the temperature of the coffee was 85 °C.

(a) State the value of A .

(1)

Initially, the coffee was cooling at a rate of 7.5 °C per minute.

(b) Find a complete equation linking H and t , giving the value of B to 3 decimal places.

(3)

(a) $H = 85$ when $t = 0$, so

$$85 = Ae^{-B(0)} + 30$$

$$85 = A + 30$$

$$A = 85 - 30 = 55$$

(b) $\frac{dH}{dt} = 55e^{-Bt}(-B) = -55Be^{-Bt}$

$\frac{dH}{dt} = -7.5$ when $t = 0$, so

$$-7.5 = -55Be^{-B(0)}$$

$$-7.5 = -55B$$

$$B = \frac{-7.5}{-55} = 0.1363\dots$$
$$= 0.136 \text{ (3dp)}$$

$$\text{so, } H = 55e^{-0.136t} + 30$$