

5. The curve  $C$  has equation  $y = f(x)$

The curve

- passes through the point  $P(3, -10)$
- has a turning point at  $P$

Given that

$$\frac{dy}{dx} = 2x^3 - 9x^2 + 5x + k$$

where  $k$  is a constant,

(a) show that  $k = 12$

(2)

(b) Hence find the coordinates of the point where  $C$  crosses the  $y$ -axis.

(3)

(a) At  $P(3, -10)$ ,  $\frac{dy}{dx} = 0$ , so

$$2(3)^3 - 9(3)^2 + 5(3) + k = 0$$

$$2(27) - 9(9) + 5(3) + k = 0$$

$$54 - 81 + 15 + k = 0$$

$$k = 81 - 54 - 15 = 12$$

(b)  $y = \int \frac{dy}{dx} dx = \int 2x^3 - 9x^2 + 5x + 12 dx$

$$= \frac{1}{2}x^4 - 3x^3 + \frac{5}{2}x^2 + 12x + c$$

at  $P(3, -10)$ ,  $-10 = \frac{1}{2}(3)^4 - 3(3)^3 + \frac{5}{2}(3)^2 + 12(3) + c$

$$-10 = \frac{81}{2} - 81 + \frac{45}{2} + 36 + c$$

$$c = -10 - \frac{81}{2} + 81 - \frac{45}{2} - 36 = -28$$

so,

$$y = \frac{1}{2}x^4 - 3x^3 + \frac{5}{2}x^2 + 12x - 28$$

crosses  $y$ -axis when  $x = 0$ , so

$$y = 0 - 0 + 0 + 0 - 28 = -28$$

Crosses at  $(0, -28)$