

6. Relative to a fixed origin O ,

- A is the point with position vector $12\mathbf{i}$
- B is the point with position vector $16\mathbf{j}$
- C is the point with position vector $(50\mathbf{i} + 136\mathbf{j})$
- D is the point with position vector $(22\mathbf{i} + 24\mathbf{j})$

(a) Show that AD is parallel to BC .

(2)

Points A , B , C and D are used to model the vertices of a running track in the shape of a quadrilateral.

Runners complete one lap by running along all four sides of the track.

The lengths of the sides are measured in metres.

Given that a particular runner takes exactly 5 minutes to complete 2 laps,

(b) calculate the average speed of this runner, giving the answer in kilometres per hour.

(4)

$$A: \begin{pmatrix} 12 \\ 0 \end{pmatrix} \quad B: \begin{pmatrix} 0 \\ 16 \end{pmatrix} \quad C: \begin{pmatrix} 50 \\ 136 \end{pmatrix} \quad D: \begin{pmatrix} 22 \\ 24 \end{pmatrix}$$

$$(a) \quad \vec{AD} = \begin{pmatrix} 22-12 \\ 24-0 \end{pmatrix} = \begin{pmatrix} 10 \\ 24 \end{pmatrix} \quad \vec{BC} = \begin{pmatrix} 50-0 \\ 136-16 \end{pmatrix} = \begin{pmatrix} 50 \\ 120 \end{pmatrix}$$

\vec{AD} and \vec{BC} are scalar multiples, because $5(\vec{AD}) = \vec{BC}$
so they are parallel.

$$(b) \quad \vec{AB} = \begin{pmatrix} 0-12 \\ 16-0 \end{pmatrix} = \begin{pmatrix} -12 \\ 16 \end{pmatrix} \quad \vec{CD} = \begin{pmatrix} 22-50 \\ 24-136 \end{pmatrix} = \begin{pmatrix} -28 \\ -112 \end{pmatrix}$$

$$\text{length 1 lap} = |\vec{AB}| + |\vec{BC}| + |\vec{CD}| + |\vec{AD}|$$

$$\begin{aligned} &= \sqrt{(-12)^2 + 16^2} + \sqrt{50^2 + 120^2} + \sqrt{(-28)^2 + (-112)^2} + \sqrt{10^2 + 24^2} \\ &= 20 + 130 + 28\sqrt{17} + 26 \\ &= 176 + 28\sqrt{17} \end{aligned}$$

$$\text{Average Speed} = \frac{\overset{(2 \text{ laps})}{2} (176 + 28\sqrt{17}) \div 1000}{\underset{(\text{min} \rightarrow \text{hr})}{5/60}} \overset{(\text{m} \rightarrow \text{km})}{=} 6.994\dots = 6.99 \text{ km/hr (3sf)}$$