

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

A curve has equation

$$x^3 + 2xy + 3y^2 = 47$$

(a) Find $\frac{dy}{dx}$ in terms of x and y

(4)

The point $P(-2, 5)$ lies on the curve.

(b) Find the equation of the normal to the curve at P , giving your answer in the form $ax + by + c = 0$, where a , b and c are integers to be found.

(3)

(a) $x^3 + 2xy + 3y^2 = 47$

$$3x^2 + \underbrace{2y + 2x \frac{dy}{dx}}_{\text{(Product Rule)}} + \underbrace{6y \frac{dy}{dx}}_{\text{(Chain Rule)}} = 0$$

$$2x \frac{dy}{dx} + 6y \frac{dy}{dx} = -3x^2 - 2y$$

$$\frac{dy}{dx} (2x + 6y) = -(3x^2 + 2y)$$

$$\frac{dy}{dx} = - \frac{3x^2 + 2y}{2x + 6y}$$

(b) At $P(-2, 5)$

$$\frac{dy}{dx} = - \frac{3(-2)^2 + 2(5)}{2(-2) + 6(5)}$$

$$= - \frac{12 + 10}{-4 + 30} = - \frac{22}{26} = - \frac{11}{13}$$

Perpendicular (Normal) gradient = $-\frac{1}{-\frac{11}{13}} = \frac{13}{11}$

$$y - 5 = \frac{13}{11} (x - (-2)) \Rightarrow \frac{13}{11} x + \frac{26}{11} - y + 5 = 0$$

$$\Rightarrow 13x + 26 - 11y + 55 = 0$$

$$\Rightarrow 13x - 11y + 81 = 0$$