

8. (a) Express $2 \cos \theta + 8 \sin \theta$ in the form $R \cos(\theta - \alpha)$, where R and α are constants,

$$R > 0 \text{ and } 0 < \alpha < \frac{\pi}{2}$$

Give the exact value of R and give the value of α in radians to 3 decimal places.

(3)

The first three terms of an arithmetic sequence are

$$\cos x \quad \cos x + \sin x \quad \cos x + 2 \sin x \quad x \neq n\pi$$

Given that S_9 represents the sum of the first 9 terms of this sequence as x varies,

(b) (i) find the exact maximum value of S_9

(ii) deduce the smallest positive value of x at which this maximum value of S_9 occurs.

(3)

$$(a) \sqrt{2^2 + 8^2} \left(\frac{2}{\sqrt{2^2 + 8^2}} \cos \theta + \frac{8}{\sqrt{2^2 + 8^2}} \sin \theta \right)$$

$$= 2\sqrt{17} \left(\frac{1}{\sqrt{17}} \cos \theta + \frac{4}{\sqrt{17}} \sin \theta \right)$$

$$= 2\sqrt{17} (\cos \alpha \cos \theta + \sin \alpha \sin \theta) \quad \cos(\theta - \alpha) = \cos \theta \cos \alpha + \sin \theta \sin \alpha$$

$$R = 2\sqrt{17} \quad \alpha = \tan^{-1} \left(\frac{4/\sqrt{17}}{1/\sqrt{17}} \right) = 1.3258, \dots \\ = 1.326 \text{ (3dp)}$$

$$(b) a = \cos x \quad d = \sin x$$

$$(i) S_9 = \frac{9}{2} (2a + (9-1)d) = \frac{9}{2} (2 \cos x + 8 \sin x)$$

$$= \frac{9}{2} (2\sqrt{17} \cos(x - 1.326))$$

Exact Max of \cos is 1,
so Exact Max of S_9 is $\frac{9}{2} (2\sqrt{17}) = 9\sqrt{17}$

(ii) $\cos(0) = 1$, so smallest positive value of x is

$$\text{when } x - 1.326 = 0 \Rightarrow x = 1.326$$