

9. The curve C has parametric equations

$$x = t^2 + 6t - 16 \quad y = 6 \ln(t + 3) \quad t > -3$$

(a) Show that a Cartesian equation for C is

$$y = A \ln(x + B) \quad x > -B$$

where A and B are integers to be found.

(3)

The curve C cuts the y -axis at the point P

(b) Show that the equation of the tangent to C at P can be written in the form

$$ax + by = c \ln 5$$

where a , b and c are integers to be found.

(4)

(a) $y = 6 \ln(t + 3) \Rightarrow \ln(t + 3) = \frac{y}{6}$

$$\Rightarrow t + 3 = e^{\frac{y}{6}} \quad \Rightarrow t = e^{\frac{y}{6}} - 3$$

Substituting,

$$x = (e^{\frac{y}{6}} - 3)^2 + 6(e^{\frac{y}{6}} - 3) - 16$$

$$x = e^{\frac{y}{3}} - 6e^{\frac{y}{6}} + 9 + 6e^{\frac{y}{6}} - 18 - 16$$

$$x = e^{\frac{y}{3}} - 25$$

$$\Rightarrow e^{\frac{y}{3}} = x + 25$$

$$\frac{y}{3} = \ln(x + 25)$$

$$y = 3 \ln(x + 25)$$

(b) When C cuts y -axis, $x = 0 \Rightarrow y = 3 \ln(0 + 25)$
 $= 3 \ln 25$
 $= 3 \ln 5^2 = 6 \ln 5$

so, P is $(0, 6 \ln 5)$

$$\frac{dy}{dx} = \frac{3}{x + 25}$$

$$\text{when } x = 0, \frac{dy}{dx} = \frac{3}{25}$$

Tangent at P is $y - 6 \ln 5 = \frac{3}{25}(x - 0)$

$$\Rightarrow \frac{3}{25}x - y = -6 \ln 5$$

$$\Rightarrow -3x + 25y = 150 \ln 5$$