

10.

$$f(x) = \frac{3kx - 18}{(x+4)(x-2)}$$

where k is a positive constant(a) Express $f(x)$ in partial fractions in terms of k .

(3)

(b) Hence find the exact value of k for which

$$\int_{-3}^1 f(x) dx = 21$$

(4)

$$(a) \frac{3kx - 18}{(x+4)(x-2)} = \frac{3kx}{(x+4)(x-2)} - \frac{18}{(x+4)(x-2)}$$

$$\frac{3kx}{(x+4)(x-2)} = \frac{A}{x+4} + \frac{B}{x-2} \Rightarrow 3kx = A(x-2) + B(x+4)$$

$$\text{when } x=2, 6k = 6B \Rightarrow B=k$$

$$\text{when } x=-4, -12k = -6A \Rightarrow A=2k$$

$$\frac{18}{(x+4)(x-2)} = \frac{C}{x+4} + \frac{D}{x-2} \Rightarrow 18 = C(x-2) + D(x+4)$$

$$\text{when } x=2, 18 = 6D \Rightarrow D=3$$

$$\text{when } x=-4, 18 = -6C \Rightarrow C=-3$$

$$f(x) = \left(\frac{2k}{x+4} + \frac{k}{x-2} \right) - \left(-\frac{3}{x+4} + \frac{3}{x-2} \right) = \frac{2k+3}{x+4} + \frac{k-3}{x-2}$$

$$(b) \int_{-3}^1 f(x) dx = \left[(2k+3) \ln|x+4| + (k-3) \ln|x-2| \right]_{-3}^1$$

$$= (2k+3) \ln 5 + (k-3) \ln |-1| - (2k+3) \ln 1 - (k-3) \ln |-5|$$

$$= (2k+3) \ln 5 + 0 - 0 - (k-3) \ln 5$$

$$= (2k+3 - k+3) \ln 5 = (k+6) \ln 5$$

$$\text{Given } (k+6) \ln 5 = 21, \quad k = \frac{21}{\ln 5} - 6$$