



Figure 1

A tank in the shape of a cuboid is being filled with water.

The base of the tank measures 20m by 10m and the height of the tank is 5m, as shown in Figure 1.

At time  $t$  minutes after water started flowing into the tank the height of the water was  $h$  m and the volume of water in the tank was  $V$  m<sup>3</sup>

In a model of this situation

- the sides of the tank have negligible thickness
- the rate of change of  $V$  is inversely proportional to the square root of  $h$

(a) Show that

$$\frac{dh}{dt} = \frac{\lambda}{\sqrt{h}}$$

where  $\lambda$  is a constant.

(3)

$$(a) \quad V = 20 \times 10 \times h = 200h \quad \frac{dV}{dh} = 200$$

$$\frac{dV}{dt} \propto \frac{1}{\sqrt{h}} \Rightarrow \frac{dV}{dt} = \frac{k}{\sqrt{h}} \quad \text{where } k \text{ is constant}$$

$$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt} = \frac{1}{200} \times \frac{k}{\sqrt{h}} = \frac{\lambda}{\sqrt{h}} \quad \text{where } \lambda = \frac{k}{200}$$

$$\left( \frac{dh}{dV} = \frac{1}{\frac{dV}{dh}} = \frac{1}{200} \right)$$

Given that

- initially the height of the water in the tank was 1.44 m
- exactly 8 minutes after water started flowing into the tank the height of the water was 3.24 m

(b) use the model to find an equation linking  $h$  with  $t$ , giving your answer in the form

$$h^{\frac{3}{2}} = At + B$$

where  $A$  and  $B$  are constants to be found.

(5)

$$(b) \quad \text{When } t=0, h=1.44$$

$$\text{When } t=8, h=3.24$$

$$\text{Separating the Variables, } \int \sqrt{h} \, dh = \int \lambda \, dt$$

$$\frac{2}{3} h^{\frac{3}{2}} = \lambda t + c$$

$$h^{\frac{3}{2}} = \frac{3\lambda}{2} t + c'$$

$$\text{When } t=0, 1.44^{\frac{3}{2}} = 0 + c' \Rightarrow c' = 1.728$$

$$\text{When } t=8, 3.24^{\frac{3}{2}} = \frac{3\lambda}{2}(8) + 1.728$$

$$5.832 = 12\lambda + 1.728$$

$$\lambda = 0.342$$

$$\text{so, } h^{\frac{3}{2}} = \frac{3(0.342)}{2} t + 1.728$$

$$= 0.513t + 1.728$$

(c) Hence find the time taken, from when water started flowing into the tank, for the tank to be completely full.

(2)

$$(c) \quad \text{When tank is full, } h=5, \text{ so}$$

$$5^{\frac{3}{2}} = 0.513t + 1.728 \Rightarrow t = \frac{5^{\frac{3}{2}} - 1.728}{0.513}$$

$$t = 18.4256\dots$$

$$t = 18 \text{ min } 26 \text{ sec}$$

(to nearest second)