

Solutions relying entirely on calculator technology are not acceptable.

- (a) Find the first three terms, in ascending powers of
- $x$
- , of the binomial expansion of

$$(3+x)^{-2}$$

writing each term in simplest form.

(4)

$$\begin{aligned} \text{(a)} \quad (3+x)^{-2} &= \left(3\left(1+\frac{x}{3}\right)\right)^{-2} = 3^{-2} \left(1+\frac{x}{3}\right)^{-2} \\ &= 3^{-2} \left(1 + (-2)\left(\frac{x}{3}\right) + \frac{(-2)(-3)}{2!} \left(\frac{x}{3}\right)^2 + \dots\right) \\ &= \frac{1}{9} \left(1 - \frac{2}{3}x + \frac{1}{3}x^2 + \dots\right) \\ &= \frac{1}{9} - \frac{2}{27}x + \frac{1}{27}x^2 + \dots \end{aligned}$$

- (b) Using the answer to part (a) and using algebraic integration, estimate the value of

$$\int_{0.2}^{0.4} \frac{6x}{(3+x)^2} dx$$

giving your answer to 4 significant figures.

(4)

$$\begin{aligned} \text{(b)} \quad \int_{0.2}^{0.4} \frac{6x}{(3+x)^2} &\approx \int_{0.2}^{0.4} 6x \left(\frac{1}{9} - \frac{2}{27}x + \frac{1}{27}x^2\right) \\ &= \int_{0.2}^{0.4} \frac{2}{3}x - \frac{4}{9}x^2 + \frac{2}{9}x^3 = \left[\frac{1}{3}x^2 - \frac{4}{27}x^3 + \frac{1}{18}x^4\right]_{0.2}^{0.4} \\ &= \frac{1}{3}(0.4)^2 - \frac{4}{27}(0.4)^3 + \frac{1}{18}(0.4)^4 - \frac{1}{3}(0.2)^2 + \frac{4}{27}(0.2)^3 - \frac{1}{18}(0.2)^4 \\ &= \frac{233}{6750} = 0.03304 \text{ (4 sf)} \end{aligned}$$

- (c) Find, using algebraic integration, the exact value of

$$\int_{0.2}^{0.4} \frac{6x}{(3+x)^2} dx$$

giving your answer in the form  $a \ln b + c$ , where  $a$ ,  $b$  and  $c$  are constants to be found.

(5)

$$\text{(c)} \quad \frac{6x}{(3+x)^2} \equiv \frac{A}{3+x} + \frac{B}{(3+x)^2}$$

$$6x \equiv A(3+x) + B = Ax + 3A + B$$

$$\text{Equating } x\text{'s, } 6 = A \Rightarrow A = 6$$

$$\text{Equating constants, } 0 = 3A + B = 3(6) + B \Rightarrow B = -18$$

$$\int_{0.2}^{0.4} \frac{6}{3+x} - \frac{18}{(3+x)^2}$$

$$= \left[6 \ln(3+x) - \frac{18}{-1} (3+x)^{-1}\right]_{0.2}^{0.4}$$

$$= \left[6 \ln(3+x) + \frac{18}{3+x}\right]_{0.2}^{0.4}$$

$$= 6 \ln(3.4) + \frac{18}{3.4} - 6 \ln(3.2) - \frac{18}{3.2}$$

$$= 6 \ln\left(\frac{3.4}{3.2}\right) - \frac{45}{136} = 6 \ln\left(\frac{17}{16}\right) - \frac{45}{136}$$