

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$2 \tan \theta (8 \cos \theta + 23 \sin^2 \theta) = 8 \sin 2\theta (1 + \tan^2 \theta)$$

may be written as

$$\sin 2\theta (A \cos^2 \theta + B \cos \theta + C) = 0$$

where A , B and C are constants to be found.

(3)

(b) Hence, solve for $360^\circ \leq x \leq 540^\circ$

$$2 \tan x (8 \cos x + 23 \sin^2 x) = 8 \sin 2x (1 + \tan^2 x) \quad x \in \mathbb{R} \quad x \neq 450^\circ \quad (4)$$

$$(a) \quad 2 \tan \theta (8 \cos \theta + 23 \sin^2 \theta) = 8 \sin 2\theta (1 + \tan^2 \theta)$$

$$\frac{2 \sin \theta}{\cos \theta} (8 \cos \theta + 23 \sin^2 \theta) = 8 \sin 2\theta (\sec^2 \theta)$$

$$16 \sin \theta + 46 \frac{\sin^3 \theta}{\cos \theta} = 8 \sin 2\theta \left(\frac{1}{\cos^2 \theta} \right)$$

$$x \cos^2 \theta: \quad 16 \sin \theta \cos \theta \cos \theta + 46 \sin \theta \cos \theta \sin^2 \theta = 8 \sin 2\theta$$

$$8 \sin 2\theta \cos \theta + 23 \sin 2\theta (1 - \cos^2 \theta) = 8 \sin 2\theta$$

$$8 \sin 2\theta \cos \theta + 23 \sin 2\theta - 23 \sin 2\theta \cos^2 \theta - 8 \sin 2\theta = 0$$

$$\sin 2\theta (8 \cos \theta + 23 - 23 \cos^2 \theta - 8) = 0$$

$$\sin 2\theta (-23 \cos^2 \theta + 8 \cos \theta + 15) = 0$$

$$\sin 2\theta (23 \cos^2 \theta - 8 \cos \theta - 15) = 0$$

$$(b) \quad \sin 2\theta (\cos \theta - 1)(23 \cos \theta + 15) = 0$$

$$\sin 2\theta = 0, \quad \cos \theta = 1, \quad \cos \theta = -\frac{15}{23}$$

$$\sin 2\theta = 0$$

$$360 \leq \theta \leq 540$$

$$720 \leq 2\theta \leq 1080$$



$$2\theta = 720, 900, 1080$$

$$\theta = 360, 450, 540$$

$$\cos \theta = 1$$



$$\theta = 360$$

$$\cos \theta = -\frac{15}{23}$$



$$\cos^{-1} \left(\frac{15}{23} \right) = 49.29 \text{ (2dp)}$$

$$\theta = 540 - 49.29 = 490.71$$

$$\text{Given } x \neq 450^\circ, \quad x = \theta = 360^\circ, 490.71^\circ, 540^\circ$$