

15. A student attempts to answer the following question:

Given that x is an obtuse angle, use algebra to prove by contradiction that

$$\sin x - \cos x \geq 1$$

The student starts the proof with:

Assume that $\sin x - \cos x < 1$ when x is an obtuse angle

$$\Rightarrow (\sin x - \cos x)^2 < 1$$

$$\Rightarrow \dots$$

The start of the student's proof is reprinted below.

Complete the proof.

(3)

Assume that $\sin x - \cos x < 1$ when x is an obtuse angle

$$\Rightarrow (\sin x - \cos x)^2 < 1$$

$$\Rightarrow \sin^2 x - 2\sin x \cos x + \cos^2 x < 1$$

$$\Rightarrow \sin^2 x + \cos^2 x - 2\sin x \cos x < 1$$

$$\Rightarrow 1 - 2\sin x \cos x < 1$$

$$\Rightarrow 2\sin x \cos x > 0$$

But, given x is obtuse,
 $\sin x > 0$ and $\cos x < 0$
 so $\sin x \cos x < 0$
 so $2\sin x \cos x < 0$

$\sin x - \cos x < 1$ for obtuse x
 gives a contradiction, so

$$\sin x - \cos x \geq 1$$