Question	Scheme	Marks	AOs
3(a)	$\mathrm{f}\left(\frac{2}{3}\right)=3\left(\frac{2}{3}\right)^3-20\left(\frac{2}{3}\right)^2+45\left(\frac{2}{3}\right)-22$ $=\frac{8}{9}-\frac{80}{9}+30-22$	M1	1.1b
	$\mathrm{f}\left(\frac{2}{3}\right)=0$ hence $(3x-2)$ is a factor of $\mathrm{f}(x)$ *	A1*	2.1
		(2)	
(b)	Begins division or factorisation so $3x^3-20x^2+45x-22=(3x-2)(x^2+...)$	M1	3.1a
	$3x^3-20x^2+45x-22=(3x-2)(x^2-6x+11)$	A1	1.1b
	$b^2-4ac=(-6)^2-4(1)(11)=\ldots$	dM1	2.1
	$b^2-4ac=-8<0$ Therefore the quadratic has no roots. Hence $\frac{2}{3}$ is the only root of the equation $\{\mathrm{f}(x)=0\}$ *	A1*	1.1b
		(4)	
(c)	$\ln 2x=\frac{2}{3}\Rightarrow 2x=\mathrm{e}^{\frac{2}{3}}\Rightarrow x=...$	M1	2.2a
	$x=\frac{1}{2}\mathrm{e}^{\frac{2}{3}}$	A1	1.1b
		(2)	
			(8 marks)

Notes:

(a)

M1: Attempts to calculate $\mathrm{f}\left(\frac{2}{3}\right)$. Attempted division of $\mathrm{f}(x)$ by $(3x-2)$ is M0.

Either line in the main scheme is acceptable.

A1*: Correct calculation, reason and conclusion. It must follow M1. Accept, for example,

$\mathrm{f}\left(\frac{2}{3}\right)=0$ hence $(3x-2)$ is a factor {of $\mathrm{f}(x)$ by the factor theorem}.

(b)

M1: Begins division or factorisation so $3x^3-20x^2+45x-22=(3x-2)(x^2\pm...)$

Requires $(3x-2)$ and the first term correct of the quadratic.

A1: $(3x-2)(x^2-6x-11)$

dM1: Considers the roots of their quadratic using discriminant or completing the square.

e.g., $b^2 - 4ac = ("-6")^2 - 4(1)("11") = \dots$ or $\left(x - \frac{b}{2}\right)^2 \pm \dots$

A1*: A correct explanation including all elements below.

- Either $b^2 - 4ac = -8 < 0$ **or** $(x-3)^2 + 2 > 0$ The inequality must be strict.
- Therefore the quadratic has no roots.
- Hence there is only one root $\left\{x = \frac{2}{3}\right\}$ of the equation $\{\mathrm{f}(x) = 0\}$

(c)

M1: Deduces that $\ln 2x = \frac{2}{3}$ and attempts to solve leading to $x = \dots$

A1: $x = \frac{1}{2}e^{\frac{2}{3}}$ o.e., e.g., $x = \frac{1}{2}\left(\sqrt[3]{e}\right)^2$