

Question	Scheme	Marks	AOs	
<b>9(a)</b>	$y = \frac{1}{x^x} \Rightarrow \ln y = \ln 1 - \ln x^x$	$y = x^{-x} \Rightarrow \ln y = \ln x^{-x}$	M1	1.1b
	$\ln y = -x \ln x^*$		A1*	2.1
			(2)	
<b>(b)</b>	$\ln y \rightarrow \frac{1}{y} \frac{dy}{dx}$		M1	1.1b
	$-x \ln x \rightarrow -\ln x - 1$		M1	1.1b
	$\frac{1}{x^{-x}} \frac{dy}{dx} = -\ln x - 1 \Rightarrow \frac{dy}{dx} = -x^{-x} (\ln x + 1)^*$		A1*	2.1
			(3)	
<b>(c)</b>	Sets $-x^{-x} (\ln x + 1) = 0$ proceeding to $x = e^{-1}$		M1	3.1a
	Attempts $y = \frac{1}{(e^{-1})^{e^{-1}}}$		dM1	1.1b
	$(e^{-1}, e^{e^{-1}})$ or $\left(\frac{1}{e}, e^{\frac{1}{e}}\right)$		A1	2.5
			(3)	
<b>(d)</b>	Either $g(x) \dots 0$ or $g(x) \dots e^{e^{-1}}$		M1	2.2a
	$0 < g(x) \leq e^{e^{-1}}$		A1	2.5
			(2)	

**(10 marks)**

**Notes:**

**(a)**

**M1:** Way 1: Takes logs of both sides and uses the subtraction law correctly;

Way 2: Rewrites  $\frac{1}{x^x}$  as  $x^{-x}$  and takes logs of both sides.

**A1\*:** Proceeds to the given result with a valid method and no errors.

Must see ln in at least one step prior to the given answer.

**(b)**

**M1:** Differentiates the left hand side implicitly to the form  $\frac{\dots dy}{y dx}$

**M1:** Differentiates the right hand side using the product rule to the form  $-\dots \ln x - \dots$

**A1\*:** Proceeds to the given result with a valid method and no errors.  $x^{-x}$  must be seen substituted before the given answer is seen unless e.g.  $\frac{dy}{dx} = -y(\ln x + 1)$  seen instead.

**(c)**

**M1:** Sets  $\{-x^{-x}\}(\ln x + 1) = 0$  proceeding to  $x = \dots$

**dM1:** Substitutes their  $x$  into  $y = x^{-x}$  or  $y = \frac{1}{x^x}$

**A1:** Correct coordinates in exact form as in the main scheme. Allow  $x = \dots$  and  $y = \dots$

**(d)**

**M1:** Either “end” found. Ignore inequalities for this mark.

**A1:**  $0 < g(x) \leq e^{e^{-1}}$  o.e. e.g.  $0 < g(x) \leq e^{\frac{1}{e}}$