Question	Scheme		Marks	AOs
11(i)(a)	$q\log_3 5 = p \Rightarrow \log_3 5^q = p$	$5 = 3^{\frac{p}{q}} \Rightarrow 5 = (3^p)^{\frac{1}{q}}$	M1	1.1b
	$5^q = 3^p$ *		A1*	2.1
			(2)	
(i)(b)	For setting up the contradiction: Assume that $\log_3 5$ is rational		B1	2.5
	Then $\log_3 5 = \dfrac{p}{q}$ and hence $5^q = 3^p$		M1	1.1b
	Concludes that this is a contradiction because e.g. powers of 5 are not multiples of 3 and states that $\log_3 5$ is irrational.		A1	2.1
			(3)	
(ii)	e.g. $a = \sqrt{2}$ and $b = 2$, $\log_{\sqrt{2}} 2 = 2$, which is rational.		M1	1.1b
	Concludes that the statement is false with clear reasoning.		A1	2.1
			(2)	
				(7 marks)

Notes:

(i)(a)

M1: Way 1: Multiplies by *q* and undoes the log correctly;

Way 2: Undoes the log correctly and rewrites $3^{\frac{p}{q}}$ as $(3^p)^{\frac{1}{q}}$

A1*: Achieves the given result with a valid argument with no steps omitted and no errors.

(i)(b)

B1: For using correct language in setting up the contradiction.

Expect to see a minimum of

- "assume" or "let" or "there is"
- $\log_3 5$ is rational or $\log_3 5 = \dfrac{p}{q}$

M1: Uses the result from part (a) to conclude $5^q = 3^p$

A1: Concludes that this is a contradiction because, e.g., powers of 5 are ot be multiples of 3, **and** states that $\log_3 5$ is irrational. Accept equivalent reasoning, e.g., 3 and 5 are coprime, unique prime factorisation.

(ii)

M1: Finds a suitable counterexample.

A1: Concludes that the statement is false. This requires:

- A valid counterexample.
- Explanation that their *a* is irrational, *b* is rational and that $\log_a b$ is rational.