

Question	Scheme	Marks	AOs
12(a)	$k \sin^2 t \cos t$	M1	1.1b
	$3 \sin^2 t \cos t$	A1	1.1b
		(2)	
(b)	$2 \sin t \cos t - \sin t = 0$	M1	3.1a
	$\sin t(2 \cos t - 1) = 0$ $\cos t = \frac{1}{2} \Rightarrow t = \dots$	$\sin t(2 \cos t - 1) = 0$ $(2x-1)\sqrt{1-x^2} = 0 \Rightarrow x = \frac{1}{2}$ $\cos t = \frac{1}{2} \Rightarrow t = \dots$	dM1 1.1b
	$t = \frac{\pi}{3}$ clearly identified	A1	1.1b
		(3)	
(c)	$\frac{dx}{dt} = -\sin t$	B1	1.1b
	$\int y \, dx = \int (\sin 2t - \sin t) "(-\sin t)" \, dt$	M1	2.1
	$\int (\sin^2 t - 2 \sin^2 t \cos t) \, dt$	A1	1.1b
	$\int \left(\frac{1}{2} - \frac{1}{2} \cos 2t - 2 \sin^2 t \cos t \right) \, dt$	dM1	3.1a
	$\int_{\frac{\pi}{3}}^0 \left(\frac{1}{2} - \frac{1}{2} \cos 2t - 2 \sin^2 t \cos t \right) \, dt = \int_0^{\frac{\pi}{3}} \left(2 \sin^2 t \cos t + \frac{1}{2} \cos 2t - \frac{1}{2} \right) \, dt$	A1	2.1
		(5)	
(d)	$\dots \sin^3 t + \dots \sin 2t - \frac{1}{2} t$	M1	1.1b
	$\frac{2}{3} \sin^3 t + \frac{1}{4} \sin 2t - \frac{1}{2} t$	A1ft	1.1b
	$\frac{2}{3} \sin^3 \frac{\pi}{3} + \frac{1}{4} \sin \frac{2\pi}{3} - \frac{\pi}{6} \{-0\}$	dM1	3.1a
	$\frac{3\sqrt{3}}{8} - \frac{\pi}{6}$	A1	2.1
		(4)	

(14 marks)

Notes:

(a)

M1: Arrives at the form $\dots \sin^2 t \cos t$ but allow e.g. $\dots \sin^2 x \cos x$

A1: $3\sin^2 t \cos t$ cao. Do not worry about what they call it. Must be in terms of t .

(b)

M1: For the key step in using $\sin 2t = 2\sin t \cos t$ to write $\{y =\} 2\sin t \cos t - \sin t$ and sets equal to 0

dM1: Attempts to factorise and

Way 1: solves for $\cos t$ and then solves for t

Way 2: replaces $\cos t$ with x and factorises, so either $(2x-1)\sqrt{1-x^2} = 0$ or $(2x-1)\sin t = 0$, solves for x and then solves for t

A1: $t = \frac{\pi}{3}$ clearly identified (other solutions should be attributed to P or S or discarded).

(c)

B1: Differentiates x with respect to t correctly, i.e., $\frac{dx}{dt} = -\sin t$

M1: For the key step in setting up the parametric integration:

$\int y dx = \int (\sin 2t - \sin t) "(-\sin t)" dt$ following through on their $\frac{dx}{dt}$

A1: Reaches $\int (\sin^2 t - 2\sin^2 t \cos t) dt$

dM1: Uses $\pm \cos 2t = \pm 1 \pm \sin^2 t$ to replace $\sin^2 t$ in the integral.

A1: Reaches $\int_0^{\frac{\pi}{3}} \left(2\sin^2 t \cos t + \frac{1}{2} \cos 2t - \frac{1}{2} \right) dt$ with no errors and with limits correctly shown in at least one previous step.

(d)

M1: Integrates their $\int_{\alpha}^{\beta} (2\sin^2 t \cos t + a \cos 2t + b) dt$ to $\frac{2}{3} \sin^3 t + \dots \sin 2t + \dots t \{+c\}$

A1ft: $\frac{2}{3} \sin^3 t + \frac{1}{4} \sin 2t - \frac{1}{2} t$ Allow $\frac{2}{3} \sin^3 t + \frac{a}{2} \sin 2t + bt$ following through on their a and b only

dM1: Substitutes in their limits and subtracts either way round.

There is no need to see the $-(0)$ from the lower limit if it is using $t=0$.

A1: $\frac{3\sqrt{3}}{8} - \frac{\pi}{6}$ cao