| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **13(a)** | $\int x\sin 3x\,\mathrm{d}x = -...x\cos 3x + ...\int \cos 3x\,\mathrm{d}x$ | M1 | 1.1b |
| | $\int x\sin 3x\,\mathrm{d}x = -...x\cos 3x + ...\sin 3x$ | dM1 | 1.1b |
| | $\int x\sin 3x\,\mathrm{d}x = -\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x\ \{+c\}$ | A1 | 1.1b |
| | | **(3)** | |
| **(b)** | Separates the variables: $\int \frac{1}{y^3}\,\mathrm{d}y = \int x\sin 3x\,\mathrm{d}x$ and<br>integrates $-\frac{1}{2}y^{-2} = -\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x + c$ | M1 | 3.1a |
| | $-\frac{1}{2}\left(\frac{1}{2}\right)^{-2} = -\frac{1}{3}\left(\frac{\pi}{6}\right)\cos\left(\frac{3\pi}{6}\right) + \frac{1}{9}\sin\left(\frac{3\pi}{6}\right) + c$ leading to $c = \ldots$ | M1 | 1.1b |
| | $...y^{-2} = \pm...x\cos 3x \pm ...\sin 3x\{+c\}$<br>$y^2 = \frac{\pm...}{\pm...x\cos 3x \pm ...\sin 3x \pm ...}$ | dM1 | 1.1b |
| | $y^2 = \frac{9}{6x\cos 3x - 2\sin 3x + 38}$ | A1 | 2.1 |
| | | **(4)** | |
| | | | **(7 marks)** |

**Notes:**

**(a)**

**M1:** Uses integration by parts the correct way round to $\int x\sin 3x\,\mathrm{d}x = -...x\cos 3x + ...\int \cos 3x\,\mathrm{d}x$

If the formula is not stated or $u = \ldots, \frac{\mathrm{d}v}{\mathrm{d}x} = \ldots$ etc. not shown then the signs must be correct to imply the method.

**dM1:** Completes the integration $-...x\cos 3x + ...\sin 3x$

**A1:** $-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x\ \{+c\}$ with or without the $+c$

**(b)**

**M1:** Separates the variables: $\int \frac{1}{y^3}\,\mathrm{d}y = \int x\sin 3x\,\mathrm{d}x$ and attempts to integrate both sides using their result from (a) which must result in the form $...y^{-2} = \pm...x\cos 3x \pm ...\sin 3x\{+c\}$

**M1:** Substitutes in the boundary conditions $x = \frac{\pi}{6}$ and $y = \frac{1}{2}$ leading to a value for $c$

May be scored following rearrangement. Note that the correct value $c = -\frac{19}{9}$

**dM1:** Makes $y^2$ the subject of the equation using correct algebra but condone slips.

Dependent on the first method mark, so the correct form is required but $c$ may not have been found.

**A1:** $y^2 = \dfrac{9}{6x\cos 3x - 2\sin 3x + 38}$ o.e. in form $y^2 = \ldots$ , e.g., $y^2 = \dfrac{0.5}{\frac{x}{3}\cos 3x - \frac{1}{9}\sin 3x + \frac{19}{9}}$ and isw.