7. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

In Harbour X, the depth of the water, H metres, at time t hours after midnight is modelled by the equation

$$H = A + B\sin\left(\frac{\pi t}{6}\right) + C\cos\left(\frac{\pi t}{6}\right) \qquad 0 \leqslant t < 24$$

where A, B and C are constants.

Given that the depth of the water is

- 8.9 m at midnight
- 4.1 m at 06:00
- 3.3 m at 09:00

(a) find a complete equation for the model.

(4)

In Harbour Y, the depth of the water, D metres, at time t hours after midnight is modelled by the equation

$$D = 6.8 + 3\sin\left(\frac{\pi t}{6}\right) + 2\cos\left(\frac{\pi t}{6}\right) \qquad 0 \leqslant t < 24$$

(b) Express D in the form $6.8 + R\sin\left(\frac{\pi t}{6} + \alpha\right)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 decimal places.

(3)

Using the answer to part (b),

(c) (i) deduce the minimum depth of the water,

(ii) find the earliest time after midnight when this minimum depth occurs. Give your answer to the nearest minute.

(3)

Harbour Y is changed to reduce the variation in the depth of the water.

(d) Explain how the equation of the model for Harbour Y could be refined to reflect this change.

(1)