

11. (i) (a) Show that if $\log_3 5 = \frac{p}{q}$, where p and q are integers, then $5^q = 3^p$ (2)

(b) Hence prove by contradiction that $\log_3 5$ is irrational. (3)

(ii) Prove by counterexample that the statement below is **false**.

“If a is irrational and b is rational, then $\log_a b$ is always irrational.”

You must make your reasoning clear.

(2)