12.

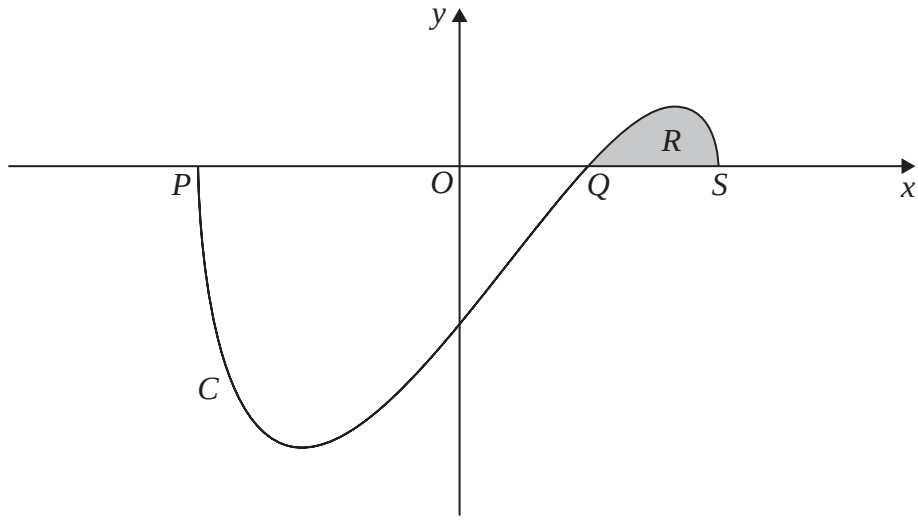

Figure 5

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

(a) Differentiate $\sin^3 t$ with respect to t

(2)

Figure 5 shows the curve C with parametric equations

$$x = \cos t \qquad y = \sin 2t - \sin t \qquad 0 \leqslant t < \pi$$

Given that C crosses the x-axis at the points $P(-1, 0)$, Q and $S(1, 0)$

(b) find the value of t at Q

(3)

The region R, shown shaded in Figure 5, is bounded by C and the x-axis between Q and S.

(c) Show that the area of R is given by

$$\int_{\alpha}^{\beta} \left(2\sin^2 t \cos t + a\cos 2t + b\right) \mathrm{d}t$$

where a, b, α and β are constants to be found and $\beta > \alpha$

(5)

(d) Using algebraic integration, find the exact area of R.
Give your answer in the form $c\sqrt{3} + d\pi$, where c and d are rational constants.

(4)