Question	Scheme	Marks	AOs
5	One of $\theta\tan 2\theta = \theta\times 2\theta$ or $1-\cos 3\theta = 1-\left(1-\frac{(3\theta)^2}{2}\right)$ or equivalents.	B1	1.1a
	$\dfrac{\theta\tan 2\theta}{1-\cos 3\theta} = \dfrac{\theta\times 2\theta}{1-\left(1-\frac{(3\theta)^2}{2}\right)}$	M1	2.1
	$=\dfrac{4}{9}$ or exact equivalent.	A1	1.1b
		(3)	
			(3 marks)

Notes

B1: Award this mark for $\theta\tan 2\theta = \theta\times 2\theta$ or $1-\cos 3\theta = 1-\left(1-\dfrac{(3\theta)^2}{2}\right)$ or equivalents.

May be seen when working on numerator or denominator separately or within the fraction.

This is a B mark so if awarding for $\cos 3\theta$ do not condone missing brackets e.g. $1-\dfrac{3\theta^2}{2}$ unless they are recovered or are implied by subsequent work.

M1: Attempts to use both correct small angle approximations in the given expression.

For this mark they must have attempted to use $\tan 2\theta = 2\theta$ and $\cos 3\theta = 1-\dfrac{(3\theta)^2}{2}$ in the given expression but condone poor bracketing e.g. $\dfrac{\theta\times 2\theta}{1-\left(1-\frac{3\theta^2}{2}\right)}$ or e.g. $\dfrac{\theta\times 2\theta}{1-1-\frac{3\theta^2}{2}}$

Do not allow e.g. $\dfrac{\theta\times 2\theta}{1-\frac{(3\theta)^2}{2}}$ as this suggests they are approximating $\dfrac{\theta\tan 2\theta}{\cos 3\theta}$

A1: Correct value. Do not allow rounded decimals e.g. 0.444 but allow if recurring decimals are clearly indicated e.g. $0.\dot{4}$ Do not allow e.g. $\dfrac{2}{4.5}$. Ignore any units if given.

Isw once a correct answer is seen.

Examples:

$\dfrac{\theta\times 2\theta}{1-\left(1-\frac{3\theta^2}{2}\right)} = \dfrac{4}{3}\left(\text{or } -\dfrac{4}{3}\right)$	scores B1M1A0 (Missing brackets not recovered)
$\dfrac{\theta\times 2\theta}{1-\frac{(3\theta)^2}{2}}$	scores B1M0A0 (Missing "1 –" in the denominator so M0)
$\dfrac{\theta\times 2\theta}{1+\left(1-\frac{(3\theta)^2}{2}\right)}$	scores B1M0A0 (Has "1 +" in the denominator so M0)
$\dfrac{\theta\times\theta}{1-\left(1-\frac{3\theta^2}{2}\right)} = \ldots$	scores B0M0A0 (The B mark could be recovered but M0 because of the incorrect numerator)
$\dfrac{\theta\times 2\theta}{1-\left(1-\frac{3\theta^2}{2}\right)} = \dfrac{2\theta^2}{\frac{9\theta^2}{2}} = \dfrac{2}{9}$	scores B1M1A0 (Missing brackets recovered)
$\dfrac{\theta\times 2\theta}{1-\left(1-\left(\frac{3\theta}{2}\right)^2\right)}$	Scores B1M0A0 (The denominator suggests an incorrect expansion – unless it was recovered.)

$\frac{\theta\times 2\theta}{\frac{9\theta^2}{2}}=\frac{2}{18}$	B1M1A0 (The B1 is awarded for the numerator but can be implied by the denominator. The M1 is implied)
$\frac{\theta\times 2\theta}{1-\left(1-\frac{3\theta^2}{2}\right)}=\frac{4}{9}$	B1M1A1 (The correct value implies correct recovery of missing brackets.)

Note that other approaches are possible using identities.
In such cases we will allow <u>correct</u> work leading to an expression that if terms in θ^3 and higher can be ignored will lead to $\frac{4}{9}$
But to score the M mark they must be using correct identities and correct approximations but condone bracketing errors as in the main scheme.

Examples:

$$\frac{\theta\tan 2\theta}{1-\cos 3\theta}=\frac{\theta\times\frac{\sin 2\theta}{\cos 2\theta}}{1-\cos 3\theta}=\frac{\theta\times\frac{2\theta}{1-\frac{(2\theta)^2}{2}}}{1-\left(1-\frac{(3\theta)^2}{2}\right)}=\frac{2\theta^2}{1-2\theta^2}\times\frac{2}{9\theta^2}=\frac{4\theta^2}{9\theta^2-18\theta^4}$$

$$=\frac{4\theta^2}{9\theta^2}=\frac{4}{9}$$

Scores B1M1A1

Similarly: $\frac{\theta\tan 2\theta}{1-\cos 3\theta}=\frac{\theta\times\sin 2\theta}{\cos 2\theta(1-\cos 3\theta)}=\frac{\theta\times 2\theta}{\left(1-\frac{(2\theta)^2}{2}\right)\left(1-\left(1-\frac{(3\theta)^2}{2}\right)\right)}=\frac{2\theta^2}{1-2\theta^2}\times\frac{2}{9\theta^2}$ etc.

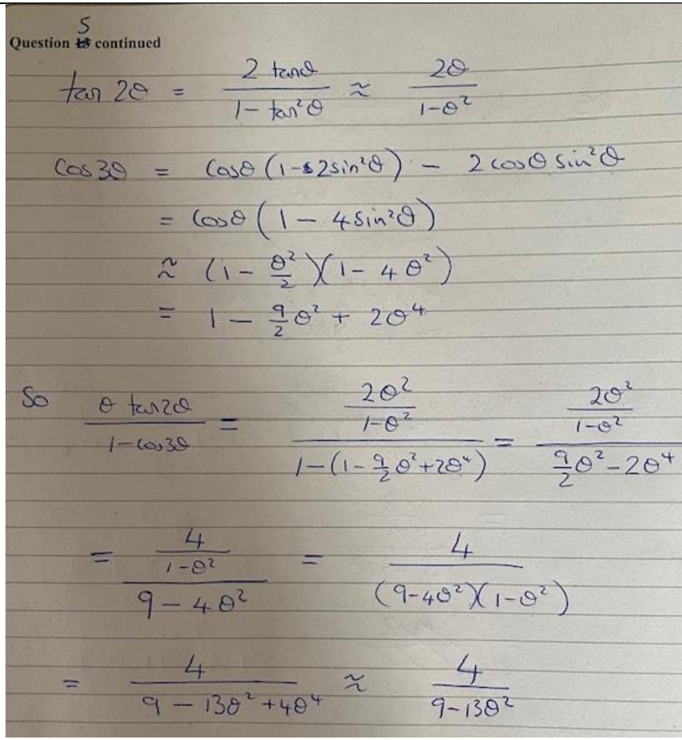
$$\frac{4\theta^2}{9\theta^2-18\theta^4}=\frac{4\theta^2}{9\theta^2}=\frac{4}{9}$$

Scores B1M1A1

$$\frac{\theta\tan 2\theta}{1-\cos 3\theta}=\frac{\theta\times\sin 2\theta}{\cos 2\theta(1-\cos 3\theta)}=\frac{\theta\times 2\theta}{\left(1-\frac{(2\theta)^2}{2}\right)\left(1-\left(1-\frac{(3\theta)^2}{2}\right)\right)}=\frac{2\theta^2}{1-2\theta^2}\times\frac{2}{9\theta^2}$$

$$=\frac{4\theta^2}{9\theta^2-18\theta^4}=\frac{4}{9-18\theta^2}=\frac{4}{9}$$

Scores B1M1A0
(They cannot just assume the term in θ^2 is 0 unless they provide a convincing limiting argument e.g. $\lim_{\theta\to 0}\frac{4}{9-18\theta^2}=\frac{4}{9}$ or equivalent)



Question ~~13~~ 5 continued

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} \approx \frac{2\theta}{1-\theta^2}$$

$$\cos 3\theta = \cos\theta(1-2\sin^2\theta) - 2\cos\theta\sin^2\theta$$

$$= \cos\theta(1-4\sin^2\theta)$$

$$\approx \left(1-\frac{\theta^2}{2}\right)(1-4\theta^2)$$

$$= 1-\frac{9}{2}\theta^2+2\theta^4$$

So

$$\frac{\theta\tan 2\theta}{1-\cos 3\theta} = \frac{\frac{2\theta^2}{1-\theta^2}}{1-\left(1-\frac{9}{2}\theta^2+2\theta^4\right)} = \frac{\frac{2\theta^2}{1-\theta^2}}{\frac{9}{2}\theta^2-2\theta^4}$$

$$= \frac{\frac{4}{1-\theta^2}}{9-4\theta^2} = \frac{4}{(9-4\theta^2)(1-\theta^2)}$$

$$= \frac{4}{9-13\theta^2+4\theta^4} \approx \frac{4}{9-13\theta^2}$$

Scores B1M1A0

$$\frac{\theta\tan 2\theta}{1-\cos 3\theta} = \frac{\theta\times\frac{2\tan\theta}{1-\tan^2\theta}}{1-\left(4\cos^3\theta-3\cos\theta\right)} = \frac{\theta\times\frac{2\theta}{1-\theta^2}}{1-\left(4\left(1-\frac{\theta^2}{2}\right)^3-3\left(1-\frac{\theta^2}{2}\right)\right)}$$

Scores B1M1A0

Note that attempts to use expansions in higher powers of θ should be sent to review.